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A two-level ILU preconditioner for electromagnetic applications☆

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ABSTRACT

Computational electromagnetics based on the solution of the integral form of Maxwell's equations with boundary element methods require the solution of large and dense linear systems. For large-scale problems the solution is obtained by using iterative Krylov-type methods provided that a fast method for performing matrix–vector products is available. In addition, for ill-conditioned problems some kind of preconditioning technique must be applied to the linear system in order to accelerate the convergence of the iterative method and improve its performance. For many applications it has been reported that incomplete factorizations often suffer from numerical instability due to the indefiniteness of the coefficient matrix. In this context, approximate inverse preconditioners based on Frobenius-norm minimization have emerged as a robust and highly parallel alternative. In this work we propose a two-level ILU preconditioner for the preconditioned GMRES method. The computation and application of the preconditioner is based on graph partitioning techniques. Numerical experiments are presented for different problems and show that with this technique it is possible to obtain robust ILU preconditioners that perform competitively compared with Frobenius-norm minimization preconditioners.

1. Introduction

The numerical solution of the Maxwell's equations [1] plays a crucial role in numerous large scale industrial and scientific applications related with electromagnetism phenomena. To name a few, the computation of the antenna radiation pattern, electromagnetic interference and compatibility studies of an electrical device with their environment, and scattering problems as the computation of the radar cross-section of a 3D body are important for aerospace industry. The performance of a computational electromagnetics (CEM) code is associated with the strengths and weaknesses of underlying numerical methods chosen for its implementation. Overall, for real-life applications the computation of an approximate solution of the linear systems arising from the discretization of the Maxwell's equations is the most demanding part in terms of computer resources. Thus, devising efficient numerical algorithms for solving these linear systems is key to develop codes capable to run with a good performance in modern computer architectures.

The most common techniques for obtaining a numerical solution of Maxwell's equations can be classified either into methods that solve the differential equations or methods that consider their integral formulation. Partial differential equations methods (PDEMs) use classical techniques like the finite-element or the finite-difference method to discretize directly the Maxwell's equations [2,3]. An advantage of these methods is that they allow the simulation of complex electrical

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structures. By contrast, the study of electrical phenomena in open domains is rather difficult and artificial boundary conditions must be imposed to simulate an infinite volume. With the rise of modern computer architectures and the sustained increment of the computational resources available, integral equations methods (IEMs) have emerged as an attractive alternative for CEM applications. These methods solve the problem by reformulating the Maxwell's equations as a set of integral equations with equivalent sources [4,5]. The integral equations relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object. This leads to a reduction on the dimensionality of the problem by one, and therefore allows significant reduction on the number of unknowns of the associated linear systems. Because the boundary conditions are incorporated into the surface integral equations, IEMs can handle general geometries in open domains without formulating any artificial boundary. Thus, they are attractive for a wide range of industrial simulations in open geometries.

The integral equations are usually discretized by means of the boundary element method (BEM) or the Method of Moments (MoM) [6,7]. Unlike PDEMs, the matrices arising from IEMs are dense and expensive to solve. Since in large-scale industrial applications the size of the matrices can be very large the application of direct gaussian elimination methods is out of context, leaving the use of Krylov-type iterative methods as the only practical alternative. The arithmetic complexity of these methods resides on the computation of matrix–vector products, operation that has a complexity of order $\mathcal{O}(n^2)$. This complexity can be reduced to $\mathcal{O}(n \log n)$ by applying optimized methods as the Fast Multipole Method [8]. An additional difficulty is that in many applications IEMs have to deal with ill-conditioned matrices that are challenging to solve, as it is the case of the matrices arising from the discretization of the electric field integral equations (EFIE).

As it is well known, the success of an iterative method for ill-conditioned problems depends on applying a suitable preconditioning technique to the system matrix. In the case of EFIE most algebraic factorized preconditioners fail to produce good converge rates or even fail to converge, see [9–12]. The best results on medium size problems were obtained with sparse approximate inverse preconditioners based on Frobenius-norm minimization [13]. Nevertheless, for large problems the relative nonzero density of the preconditioner is too small with a negative effect on the performance of this class of preconditioners. These problems may be overcome by performing spectral low-rank updates (SLRU) of the preconditioned matrix [14–16]. This technique consists in shifting by one a subset of the smallest eigenvalues that play a key role in slowing down the convergence of Krylov methods. The results of the numerical experiments show that the SLRU technique can improve considerably the performance, specially when multiple right hand sides have to be solved as is the case for scattering problems. Alternative techniques implement flexible variants of the GMRES method [17].

Our aim in this work is to present a technique for the computation of ILU-type preconditioners for ill-conditioned CEM applications. The method is based on graph partitioning techniques applied to the near field matrix of the linear systems. The paper is organized as follows. In Sections 2 and 3 we review the main ideas of graph partitioning, and the algorithm for computing a two-level ILU for CEM applications is presented. Then, the numerical results are presented in Section 4. Finally, the main conclusions are outlined in Section 5.

2. Graph partitioning

Graph partitioning is a widely used technique in parallel processing as it provides an effective way to distribute unstructured computations among processors. This decomposition is achieved by splitting the adjacency graph of a matrix into p parts subject to some constraints. Although a number of different methods have been proposed in the literature [18], the idea behind graph partitioning is the computation of a p-way partitioning of the graph keeping the size of the p subgraphs balanced while minimizing to some extent the number of edges that are cut. Let us describe briefly the technique.

Let A be a sparse structurally symmetric matrix. The associated undirected adjacency graph $G = (V, E)$ consists of a set nodes $V = \{1, \ldots, n\}$, one node for each row or column of the matrix, and the edge set E. There is an edge $\langle i, j\rangle$ for any matrix entry $a_{ij} \neq 0$. Note that there is no distinction between $\langle i, j\rangle$ and $\langle j, i\rangle$. For nonsymmetric sparse patterns the adjacency graph of $A + A^T$ is considered instead. We define the separator set as the group of nodes which are connected by edges that are cut in the graph partition. We also define the group of interior nodes as the nodes which are connected with the separator set. Thus, there are p groups of interior nodes, one for each subgraph of the partition.

By numbering first the interior nodes and taking the separator set last, the matrix is permuted into the following block angular form:

$$P^T A P = \begin{pmatrix} A_1 & & & & B_1 \\ & A_2 & & & B_2 \\ & & \ddots & & \vdots \\ & & & A_p & B_p \\ C_1 & C_2 & \cdots & C_p & A_S \end{pmatrix} \tag{1}$$

where P is a permutation matrix. The diagonal blocks $A_1, \ldots, A_p$ correspond to subgraphs induced by the interior nodes in the graph decomposition, the off-diagonal blocks B_i *y* C_i represent the connections between interior nodes and the separator set, and A_s correspond to the subgraph induced by the separator set. By computing an incomplete factorization for a matrix structured as in (1) one may obtain an efficient preconditioner for solving linear systems iteratively.