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A new technique to estimate the risk-neutral processes in jump–diffusion commodity futures models

L. Gómez-Valle, Z. Habibilashkary, J. Martínez-Rodríguez *

Departamento de Economía Aplicada, Facultad de Ciencias Económicas y Empresariales, Universidad de Valladolid, Avenida del Valle de Esgueva 6, 47011, Valladolid, Spain

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ABSTRACT

In order to price commodity derivatives, it is necessary to estimate the market prices of risk as well as the functions of the stochastic processes of the factors in the model. However, the estimation of the market prices of risk is an open question in the jump–diffusion derivative literature when a closed-form solution is not known. In this paper, we propose a novel approach for estimating the functions of the risk-neutral processes directly from market data. Moreover, this new approach avoids the estimation of the physical drift as well as the market prices of risk in order to price commodity futures. More precisely, we obtain some results that relate the risk-neutral drifts, volatilities and parameters of the jump amplitude distributions with market data. Finally, we examine the accuracy of the proposed method with NYMEX (New York Mercantile Exchange) data and we show the benefits of using jump processes for modelling the commodity price dynamics in commodity futures models.

1. Introduction

The behaviour of many commodity futures has become highly unusual over the past decades. Prices have experienced significant run-ups, and the nature of their fluctuations has changed considerably. This is partly due to financial firms with no inherent exposure to the commodities have adopted strategies of portfolio diversification into commodity futures markets as an asset class, see [1]. However, energy commodities are different from financial assets such as equity and fixed-income securities. For example, changes in market expectations, or even unanticipated macroeconomic developments may cause sudden jumps in energy prices, see [2]. Therefore, traditional modelling techniques are not directly applicable.

In order to price commodity derivatives, the empirical features of the commodity prices need to be considered. First, the spot price and other factors were assumed to follow diffusion processes. For example, Gibson and Schwartz [3] assumed that the spot price and the convenience yield were mean-reverting diffusion processes. Then, Schwartz [4] reviewed one and two-factor models and developed a three-factor mean-reverting diffusion model. Later, Miltersen and Schwartz [5] also considered a three-factor model in order to price commodity futures and futures options. More recently, in the literature, jump–diffusion models have been considered because there are numerous empirical studies which show that commodity prices exhibit jumps, [6,7] and so on. Hilliard and Reis [8] considered a three-factor model where the spot price follows a jump–diffusion stochastic process. Yan [9] extended existing commodity valuation models to allow for stochastic volatility and simultaneous jumps in the spot price and volatility. Hilliard and Hilliard [10] used the standard geometric Brownian

* Corresponding author.
E-mail addresses: lourdes@eco.uva.es (L. Gómez-Valle), ziba.habibilashkary@alumnos.uva.es (Z. Habibilashkary), julia@eco.uva.es (J. Martínez-Rodríguez).

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motion augmented by jumps to describe the underlying spot and mean-reverting diffusions for the interest rate and convenience yield state variables for gold and copper prices.

In this paper, we consider a two-factor jump–diffusion commodity model, where one of the factors is the commodity spot price. In the commodity literature, it is very common to use affine models for its simplicity and tractability. They select the simple parametric functions for the model in order to obtain a closed-form solution for the pricing problem. This is mainly important for the market prices of risk, which are assumed to be constant in most of the cases. Then, all the functions can be easily estimated and the commodity derivatives priced. However, there is not any empirical evidence either consensus about affine models are the best models to price commodity futures. Furthermore, the market prices of risk are not observed in the markets. If we considered other more realistic functions for the state variables or the market prices of risk or even a nonparametric approach, then, the model would not be affine anymore, a closed-form solution could not be obtained and therefore, the estimation of the market prices of risk would not be possible. In fact, this last problem is an open question in the jump–diffusion commodity literature.

The main contribution of this paper is twofold. First, we obtain some results that allow us to estimate the risk-neutral functions of a two-factor jump–diffusion commodity model directly from commodity spot and futures data on the markets. Therefore, we can obtain a closed-form solution or a numerical approximation for the pricing problem without estimating the market prices of risk, which are not observed and possible to estimate when a closed-form solution is not known. Second, we show the effect of considering jumps in the commodity spot price over the futures prices. We use NYMEX data and a nonparametric approach to estimate the whole functions of our two-factor model. We think that using a nonparametric approach is more realistic than using an affine model.

The remaining of the paper is arranged as follows. In Section 2, we present a two-factor jump–diffusion model to price commodity futures. In Section 3, we prove some results which allow us to estimate the risk-neutral drift, jump intensity and parameters of the distribution of the jump amplitude from spot commodity price and futures data. In Section 4, we estimate our two-factor jump–diffusion model with NYMEX data by means of a nonparametric approach, we price commodity futures and we show its supremacy over a diffusion model. Section 5 concludes.

2. The valuation model

In this section, we present a two-factor commodity futures model. The first factor is the spot price S, and the second factor is δ, which could be, for example, the instantaneous convenience yield or the volatility among other possible variables. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space equipped with a filtration $\mathcal{F}$ satisfying the usual conditions, see [11,12] or [13]. The factors of the model are assumed to follow this joint jump–diffusion stochastic process:

$$dS(t) = \mu_S(S(t), \delta(t))dt + \sigma_S(S(t), \delta(t))dW_S(t) + J(S(t), \delta(t), Y(t))dN(t), \tag{1}$$

$$d\delta(t) = \mu_\delta(S(t), \delta(t))dt + \sigma_\delta(S(t), \delta(t))dW_\delta(t), \tag{2}$$

where μ_S and μ_δ are the drifts, σ_S and σ_δ the volatilities. The jump amplitude J is a function of the two factors and Y which is a random variable with probability distribution Π. Moreover, W_S and W_δ are Wiener processes and N represents a Poisson process with intensity λ. We assume that the standard Brownian motions are correlated with:

$$Cov(W_S, W_\delta) = \rho t.$$

However, W_S and W_δ are assumed to be independent of N. We also assume that the jump magnitude and the jump arrival time are uncorrelated with the diffusion parts of the processes. We suppose that the functions $\mu_S, \mu_\delta, \sigma_S, \sigma_\delta, J, \lambda$ and Π satisfy suitable regularity conditions: see [11,14]. Under the above assumptions, a commodity futures price at time t with maturity at time T, $t \leq T$, can be expressed as $F(t, S, \delta; T)$ and at maturity it is

$$F(T, S, \delta; T) = S.$$

Finally, we assume that there exists a replicating portfolio for the futures price and then, the futures price can be expressed by

$$F(t, S, \delta; T) = E^{\mathcal{Q}}[S(T)|S(t) = S, \delta(t) = \delta], \tag{3}$$

where $E^{\mathcal{Q}}$ denotes the conditional expectation under the $\mathcal{Q}$ measure which is known as the risk-neutral probability measure. The two-factor model (1)–(2) under $\mathcal{Q}$ measure is as follows:

$$dS = \left(\mu_S - \sigma_S\theta^{W_S} + \lambda^{\mathcal{Q}} E_Y^{\mathcal{Q}}[J]\right) dt + \sigma_S dW_S^{\mathcal{Q}} + Jd\tilde{N}^{\mathcal{Q}}, \tag{4}$$

$$d\delta = \left(\mu_\delta - \sigma_\delta\theta^{W_\delta}\right) dt + \sigma_\delta dW_\delta^{\mathcal{Q}}, \tag{5}$$

where $W_S^{\mathcal{Q}}$ and $W_\delta^{\mathcal{Q}}$ are the Wiener processes under $\mathcal{Q}$ and $Cov(W_S^{\mathcal{Q}}, W_\delta^{\mathcal{Q}}) = \rho t$. The market prices of risk of Wiener processes are $\theta^{W_S}(S, \delta)$ and $\theta^{W_\delta}(S, \delta)$, and $\tilde{N}^{\mathcal{Q}}$ represents the compensated Poisson process, under $\mathcal{Q}$ measure, with intensity $\lambda^{\mathcal{Q}}(S, \delta) = \lambda(S, \delta)\theta^N(S, \delta)$.