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A direct LU solver for pricing American bond options under Hull–White model

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ABSTRACT

The main goal of this paper is to propose a novel numerical algorithm to price American options on bonds. For this purpose, we illustrate the performance of this method by means of the valuation of an American Put Option on a discount bond under the extended Vasicek model due to Hull and White (HW) and using the consistent forward rate curves. In particular, an implicit Crank–Nicolson (CN) scheme in time is applied obtaining a discretized linear complementarity problem (LCP) and then we introduce a direct LU based method to solve the LCP. Finally, we carry out numerical experiments to examine the convergence of this method and to testify the efficiency and effectiveness of this numerical scheme against other standard approaches.

1. Introduction

The present work analyzes in depth the valuation of American options on zero-coupon bond using the one-factor Hull–White model (see [1, pp. 577–581] and [2, Chap. 3, pp. 73–80] for an excellent survey).

Market models have recently emerged as a market standard for the pricing of exotic interest rate products. In these models the dynamics of Libor and swap rates are directly specified. In spite of their increasing popularity, market models have one serious drawback. An accurate implementation can only be made through simulation, which is typically slow due to the large number of discrete tenor market rates needed to be evolved through time. This problem may be acute for products with an early exercise provision, such as American-style bond options, as simulation is not naturally suited for performing backward-in-time calculations needed to determine the optimal exercise strategy.

In contrast to market models, the Hull–White model incorporates a few appealing properties. First, it is analytically tractable.

Second, path-independent products with an early exercise provision can also be efficiently evaluated by using finite differences methods.

The computational efficiency offered by the Hull–White model explains why many banks still use this kind of model for market to market, risk calculation and other management purposes even after the introduction of the market models. Despite its popularity, rigorous studies which compare different alternatives of its numerical implementation by means of pure deterministic methods, such as the finite difference methods, are scarce.

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Vetzal [3] made a case that using implicit methods could improve upon the performance of the more traditionally used trinomial-tree based techniques. While this is handy, it is unclear how Vetzal's implicit finite differences approach actually considers boundary conditions and to what extent this choice may affect the convergence to the correct solution in some specific valuation problems. In fact, Vetzal's alternative discretization strategy, is essentially the same as the utilized in [4,5] for numerical implementations of the explicit-type in order to avert the requirement of the specification of *spatial* boundary conditions. However, it can be proved for all three approaches that is indeed equivalent to impose the discretized version of the PDE in the boundary, a technical point that is difficult to explain from PDE literature perspective and even worst, rather nonsensical from a theoretical point of view.

More recently, in [6] a Crank–Nicolson method for valuing interest rate derivatives is proposed for some specific consistent models.

Although the method extends the seminal methodology [4,7] of Hull and White allowing the use of implicit methods, again the apparent lack of boundary conditions is definitely a sign of a heterodox use of the discretized version of each PDE at the boundary. Moreover, in order to deal with the Kolmogorov Forward Equation (henceforth KFE), the forward induction approach introduced by Jamshidian [8] is adapted to be suitable for a pure implicit scheme, unfortunately this feature adds computational costs to the whole numerical method. Last but not least, none of these works have the solution to the early-exercise problem when implicit methods are taking into account.

Due to the complexity of free-boundary problems, we rely on numerical experiments. We first reduce the Hull–White PDE after some transformations with the help of a reference variable. Second, as finite difference methods are straightforward to implement we compare explicit and Crank–Nicolson methods with the more traditional lattice-based approaches for European options. Finally, we consider American-style bond options and the linear complementarity problem (LCP) related with their valuation. For the Crank–Nicolson implementations, we solve the LCP using LU decomposition and a modified backward substitution with a projection. Experiments show that the direct LU algorithm is much faster and robust than the projected SOR method. The overall numerical results show that a Crank–Nicolson implementation is efficient and robust outperforming the rest.

The outline of the work is as follows. In Section 1 we briefly review the Hull–White model.

Next, in Section 2, we transform the Hull–White PDE into a more simple diffusion equation. In Section 3, we describe the application of the Finite Difference θ-scheme to this reduced PDE. In Section 4, the numerical results, for both explicit and Crank–Nicolson methods, are presented and compared with the closed forms solutions (European-style options) and those obtained with the lattice method developed in [4]. Later, a direct LU algorithm for the complementarity problem resulting from pricing American-style options and linked numerical experiments are provided. Finally, Section 5 contains conclusions.

2. The model

We recall the *risk neutral* dynamics of the Hull–White model (henceforth HW):

$$dr = [\Phi(t) - ar]\,dt + \sigma\,dW^{\mathbb{Q}} \tag{1}$$

where r denotes the short rates evolution process, a and σ are the model parameters and W denotes a Wiener process under the risk neutral measure $\mathbb{Q}$.

It is well known that the price $P(r,t,S)$ at time t of a pure discount bond with face value 1 monetary unit at its maturity date S is given as follows

$$P(r,t,S) = A(t,S)e^{-B(t,S)r}, \tag{2}$$

where

$$B(t,S) = \frac{1 - e^{-a(S-t)}}{a}, \tag{3}$$

and[1]

$$\log A(t,S) = \log\frac{P(0,S)}{P(0,t)} - B(t,S)\partial_t \log P(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aS} - e^{-at})^2(e^{2at} - 1). \tag{4}$$

Let $V = V(r,t)$ be the value of a contingent claim on a zero-coupon bond where the holder can receive a given payoff $g(r,t)$ at the expiry date $t = T$. The option pricing problem can be formulated as the following PDE

$$\begin{cases} \dfrac{\sigma^2}{2}\partial_{rr}V + (\Phi(t) - ar)\,\partial_r V - rV + \partial_t V = 0, & (r,t) \in \mathbb{R}\times]0,T[, \\ V(r,T) = g(r,T), & r \in \mathbb{R}. \end{cases} \tag{5}$$

[1] $\partial_x = \frac{\partial}{\partial x}$ is the partial derivative with respect to x.