Journal of Computational and Applied Mathematics 309 (2017) 493–510

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Numerical solution of fractional pantograph differential equations by using generalized fractional-order Bernoulli wavelet

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ARTICLE INFO

Article history:
Received 3 December 2015
Received in revised form 4 May 2016

Keywords:
Generalized fractional-order Bernoulli wavelet
Fractional pantograph differential equations
Caputo derivative
Operational matrix
Numerical solution
Collocation method

ABSTRACT

In the current study, new functions called generalized fractional-order Bernoulli wavelet functions (GFBWFs) based on the Bernoulli wavelets are defined to obtain the numerical solution of fractional-order pantograph differential equations in a large interval. For the concept of fractional derivative we will use Caputo sense by using Riemann–Liouville fractional integral operator. First, the generalized fractional-order Bernoulli wavelets are constructed. Then, these functions and their properties are employed to derive the GFBWFs operational matrices of fractional integration and pantograph. The operational matrices of integral and pantograph are utilized to reduce the problem to a set of algebraic equations. Finally, some examples are included for demonstrating the validity and applicability of our method.

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1. Introduction

Fractional calculus is increasingly used in various areas of physics and engineering, namely in electromagnetism [1], visco-elastic materials [2], fluid mechanics [3], dynamics of viscoelastic materials [4], continuum and statistical mechanics [5] and propagation of spherical flames [6]. However, since the kernel of these differential equations is fractional, it is very difficult to obtain their exact solutions. Therefore, extensive research has been performed on the development of numerical methods for fractional differential equations. These methods include Chebyshev collocation method [7], Laplace transform method [8], differential transform method [9,10], Adomian decomposition method [11], Legendre operational matrix [12], CAS wavelet [13], etc.

In this paper, we consider the fractional pantograph differential equation

$$D^{\gamma}u(t) = a(t)u(t) + \sum_{r=1}^{l} b_r(t)D^{\gamma_r}u(q_r t), \quad m-1 < \gamma \le m,\ t \in [0, h], \tag{1}$$

subject to the initial conditions

$$u^{(i)}(0) = \mu_i, \quad i = 0, 1, \ldots, m-1. \tag{2}$$

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http://dx.doi.org/10.1016/j.cam.2016.06.005
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Here, $0 < q_r < 1,\ 0 \le \gamma_r < \gamma \le m,\ r = 1, 2, \ldots, l$; u is an unknown function; a and $b_r, r = 1, 2, \ldots, l$, are the known functions defined in $[0, h]$.

Fractional delay differential equations (FDDEs) have many applications in different technical systems, such as automatic control, biology and hydraulic networks, long transmission lines, economy and biology [14]. The pantograph equation is one of the most important kinds of delay differential equations, and plays an important role in explaining various phenomena. The pantograph delay differential equations arise in many applications namely electrodynamics. In recent years, several numerical methods have been devoted to solve of pantograph delay differential equations of integer order such as, Chebyshev polynomials [15], Bernoulli polynomials [16], variational iteration method [17], etc. But there are few works devoted to numerical solution of pantograph delay differential equations of fractional order. From these works we can mention, Hermite wavelet method [18], spectral-collocation method [19] and Legendre multiwavelet collocation method [20].

Wavelet theory is a relatively new and an emerging area in mathematical research. It has been applied in a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for waveform representations and segmentations, time–frequency and analysis of fast algorithms for easy implementation [21]. Wavelets permit the accurate representation of a variety of functions and operators [22]. Moreover, wavelets establish a connection with fast numerical algorithms [23]. For solving fractional order differential equations, the operational matrices of fractional order integration for the Haar wavelet [24], Chebyshev wavelet [25], Legendre wavelet [26], CAS wavelet [27] and Bernoulli wavelet [28] are calculated.

Recently, the authors [29] defined the new orthogonal functions based on the shifted Legendre polynomials to obtain numerical solution of fractional-order differential equations. The paper [30] extended this definition and presented the operational matrix of fractional derivative and integration for such functions to construct a new Tau technique for solving fractional partial differential equations (FPDEs). The authors [31] proposed the fractional-order generalized Laguerre functions based on the generalized Laguerre polynomials. They used these functions to find numerical solution of systems of fractional differential equations. The paper [32], presented a collocation method based on the Bernstein polynomials for the fractional Riccati type differential equations. Moreover, the authors of [33] expanded the fractional Legendre functions to interval $[0, h]$ and to acquire numerical solution of FPDEs.

So, our purpose is to construct the new fractional-order wavelets based on Bernoulli wavelets on the interval $[0, h]$ for solving fractional pantograph differential equations.

In this work, firstly the various terms in the underlying fractional pantograph differential equation are approximated by linear combinations of the generalized fractional-order Bernoulli wavelets and truncating them at optimal levels. Finally, the problem is converted to an algebraic equation by introducing the GFBWFs operational matrices of fractional integration and pantograph. Therefore, there are some questions to be answered:

(i) How to derive the GFBWFs operational matrices of fractional integration and pantograph.
(ii) How to analyze the fractional pantograph differential equations via the GFBWFs operational matrices of fractional integration and pantograph.
(iii) How to select value of fractional order (α) of new functions for different problems.

The present article is organized as follows. In Section 2, we introduce some basic definitions of fractional calculus. In Section 3, we construct the GFBWFs and the generalized fractional-order Bernoulli functions (GFBFs) and give their properties. In Section 4, the GFBWFs operational matrices of fractional integration and pantograph are obtained. In Section 5, the numerical method for solving the fractional pantograph differential equations is expressed. In Section 6, we obtain an error function. This function can be checked the accuracy of the approximate solutions for the values of γ, that the exact solution is not known. In Section 7, we report our numerical results and demonstrate the accuracy of the proposed method by considering numerical examples. A conclusion is given in Section 8.

2. Basic definitions of fractional calculus

In this section, we recall the essentials of the fractional calculus theory that will be used in this paper.

Definition 1. The Riemann–Liouville fractional integral operator of order γ is defined as [28]

$$I^{\gamma}u(x) = \begin{cases} \dfrac{1}{\Gamma(\gamma)} \displaystyle\int_0^x \dfrac{u(s)}{(x-s)^{1-\gamma}} ds, & \gamma > 0,\ x > 0, \\ u(x), & \gamma = 0. \end{cases} \tag{3}$$

For the Riemann–Liouville fractional integral we have [28,32]

1. $I^{\gamma_1} I^{\gamma_2} u(x) = I^{\gamma_1+\gamma_2} u(x)$,
2. $I^{\gamma}(\lambda_1 u(x) + \lambda_2 y(x)) = \lambda_1 I^{\gamma} u(x) + \lambda_2 I^{\gamma} y(x)$,
3. $I^{\gamma} x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\gamma+1)} x^{\gamma+\beta},\ \beta > -1$,

where $\gamma_1, \gamma_2, \lambda_1$ and λ_2 are constants.

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