Journal of Computational and Applied Mathematics 309 (2017) 674–682

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A new approach for ranking fuzzy numbers based on possibility theory☆

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ARTICLE INFO

Article history:
Received 23 October 2015
Received in revised form 15 May 2016

Keywords:
Generalized fuzzy numbers
Ranking
Triangular fuzzy numbers
Trapezoidal fuzzy numbers

ABSTRACT

Most of the ranking methods discuss only triangular fuzzy numbers or trapezoidal fuzzy numbers. In this paper, we will introduce a new method of ranking generalized $L - R$ fuzzy numbers based on possibility theory and the implication of possibilistic mean and possibilistic standard deviation. Normal fuzzy numbers are the exceptional cases. The calculation of the proposed method is simpler and easier. Some comparative examples are used to illustrate the advantages of the proposed method.

1. Introduction

Ranking fuzzy numbers has a remarkable role in approximate reasoning, decision making, optimization, forecasting and some other fuzzy application systems. The concept of ranking fuzzy numbers was initiated by Jain [1] in 1976. The methods for ranking fuzzy numbers are classified into four major classes such as: (1) preference relation: [2–16]; (2) fuzzy mean and spread: probability distribution [12,13]; (3) fuzzy scoring: proportional to optimal [6], left or right scores [1,17,18], central index [19,20,14], area measurement [21]; (4) linguistic expression: intuition [13,15], linguistic approximation [22].

In the methods mentioned above, the membership function of fuzzy numbers is assumed to be normal. But the normal membership function is not adequate in many cases. In recent years, many papers investigated generalized fuzzy numbers. The ranking order is based on the generalized fuzzy numbers, in which normal fuzzy numbers are the exceptional case. Cheng [23] proposed ranking fuzzy numbers with function of synthetic index $R = \bar{x}^2 + \bar{y}^2$ and the concept of variation coefficient in 1998. Chu et al. [24] found the defect of judge standard and modified the ranking method with the function of index $S = \overline{xy}$ in 2002. However, Wang et al. [25] firstly pointed out that the centroid formulae given by Cheng [23] was wrong, and he gave a correct centroid formulae of generalized fuzzy numbers. In 2008, Wang [26] also presented the inaccuracy of ranking method by Cheng [23] and Chu et al. [24], and proposed a modified method for ranking fuzzy number with the size of area between centroid and original point. From then on, Zhao [27] gave a new ranking index based on the center of fuzzy numbers. Zeng et al. [28] gave a new ranking method based on the centroidal and degree of fragmentation of fuzzy numbers. Zeng et al. [29] presented the index method of ranking fuzzy numbers. It can be seen that the methods for ranking fuzzy numbers are based on the centroid in recent years. In 2013, Veeramani [30] listed several ranking methods and proposed a direct formula for ordering the generalized fuzzy number based on weighted expected value.

☆ The project is supported by NNSF (No. 91420202) and Youth Foundation of Weifang City.
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http://dx.doi.org/10.1016/j.cam.2016.05.017
0377-0427/

In this paper, a new method will be proposed based on possibility theory. Fuzzy set theory was proposed by L.A. in 1978. It is the foundation that possibility theory is significantly rooted. With the possibility theory, C. Carlsson et al. [31] gave the definitions of possibilistic mean value and possibilistic standard deviation about normal fuzzy numbers. We propose the definitions of possibilistic mean value and possibilistic standard deviation about generalized fuzzy numbers, normal fuzzy numbers are the exceptional cases. Based on possibility theory and implication of possibilistic mean and possibilistic standard deviation, we rank the generalized $L-R$ fuzzy numbers. The rest of the paper is organized as follows: In Section 2, new defines about generalized $L-R$ fuzzy numbers, possibilistic mean of generalized $L-R$ fuzzy numbers and possibilistic variance of generalized $L-R$ fuzzy numbers are introduced. In Section 3, a new method for ranking generalized $L-R$ fuzzy numbers, generalized triangle fuzzy numbers and generalized trapezoid fuzzy numbers is proposed. In Section 4, numerical examples prove the feasibility of the ranking order. Section 5 concludes the whole paper.

2. Preliminaries

In this section, based on the notions of fuzzy numbers and the definitions about possibilistic mean, possibilistic variance and possibilistic standard deviation of fuzzy numbers, we will give the definitions of generalized $L-R$ fuzzy numbers and the mean, variance of generalized fuzzy numbers.

Definition 2.1 (*[26]*)**.** Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \to [0, 1]$, where $\mu_A(x)$, $\forall x \in U$, indicates the degree of x in A. A is normal iff $\sup_{x\in U} \mu_A(x) = 1$. A is convex iff $\mu_A(\lambda x + (1-\lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, where $\wedge$ denotes the minimum operator. A is a fuzzy number iff A is normal and convex on U.

Definition 2.2 (*[26,32]*)**.** A generalized fuzzy number $\tilde{A}$ is described as any fuzzy subset of the universe set U with membership function $\mu_{\tilde{A}}$ defined as follows:

(a) $\mu_{\tilde{A}}$ is a continuous mapping from U to the closed internal $[0, \omega]$, $0 < \omega \leq 1$;
(b) μ_A^L is strictly increasing on $(-\infty, a]$;
(c) $\mu_{\tilde{A}}(x) = \omega$, for all $x \in [a, b]$, as ω is a constant and $0 < \omega \leq 1$;
(d) μ_A^R is strictly decreasing on $[b, +\infty)$.

Definition 2.3. The membership function $\mu_{\tilde{A}}$ of generalized fuzzy number $\tilde{A}$ can be expressed by

$$\mu_{\tilde{A}} = \begin{cases} \omega\mu_A^L(x), & x \leq a, \\ \omega, & a \leq x \leq b, \\ \omega\mu_A^R(x), & x \geq b, \end{cases}$$

where $\omega\mu_A^L : [a, b] \to [0, \omega]$ and $\omega\mu_A^R : [c, d] \to [0, \omega]$.

Based on the basic theories of fuzzy numbers, $\tilde{A}$ is a normal fuzzy number if $\omega = 1$, whereas $\tilde{A}$ is a nonnormal fuzzy number if $0 < \omega < 1$.

Especially, the generalized fuzzy number $\tilde{A}$ is called generalized $L-R$ fuzzy number when $\mu_A^L = L(\frac{a-x}{m})$, $x \leq a$; $\mu_A^R = R(\frac{x-b}{n})$, $x \geq b$.

Remarks. 1. For measuring the average level of generalized $L-R$ fuzzy numbers, based on the definition of [31], we will give the definition of its possibilistic mean.
2. For measuring the deviate degree from average level of generalized $L-R$ fuzzy numbers, based on the definition of [31], we will propose the definition of the possibilistic variance and the possibilistic standard deviation.

Definition 2.4. For a generalized $L-R$ fuzzy number $\tilde{A}$, we call $M(\tilde{A})$ the possibilistic mean value of generalized $L-R$ fuzzy numbers, if

$$M(\tilde{A}) = \frac{\int_0^\omega \lambda \frac{a_1(\lambda)+a_2(\lambda)}{2} d\lambda}{\int_0^\omega \lambda d\lambda} = \frac{1}{\omega^2}\int_0^\omega \lambda[a_1(\lambda) + a_2(\lambda)]d\lambda. \tag{1}$$

Definition 2.5. For a generalized $L-R$ fuzzy number $\tilde{A}$, we define

$$V(\tilde{A}) = \frac{1}{2}\int_0^\omega \lambda(a_2(\lambda) - a_1(\lambda))^2 d\lambda, \tag{2}$$

$$\sigma(\tilde{A}) = \sqrt{V(\tilde{A})} = \sqrt{\frac{1}{2}\int_0^\omega \lambda(a_2(\lambda) - a_1(\lambda))^2 d\lambda}. \tag{3}$$

They are called possibilistic variance, possibilistic standard deviation of $\tilde{A}$.