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An offline/online algorithm for a class of stochastic multiple obstacle scattering configurations in the half-plane

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ABSTRACT

Efficient computational models are required to understand quantities of interest (QoI) such as the intensity of sound radiated by non-deterministic configurations (comprising multiple obstacles) in the half-plane. The stochastic nature of the configurations requires tens of thousands of realizations to compute the expected value and standard deviation of the random QoI. The acoustic wave scattering configuration usually comprises multiple deterministic structures, whose locations in the half-plane are non-deterministic. Thus understanding the nature of the deterministic structures is intrinsically an offline process, whilst understanding the nature of the resulting QoI is an online process. The online process includes interactions between the structures and the half-plane boundary. In accordance with the physical offline/online process, we develop a computational T-matrix based offline/online reduced order model for large realizations of stochastic acoustic wave scattering configurations in the half-plane. We present results with more than 60 000 Monte Carlo (MC), quasi-MC (QMC) and generalized Polynomial Chaos (gPC) simulations, which are evaluated using our efficient online scheme for the acoustic scattering half plane model. Our offline/online deterministic and stochastic framework can be used in conjunction with any numerical method for the offline simulations of scattering from a single obstacle in free-space.

1. Introduction

In this article we develop and demonstrate an efficient offline/online stochastic algorithm for simulating scattering of time-harmonic acoustic waves by an uncertain configuration of N distinct particles located in the upper half-plane bounded by a sound soft wall along the x-axis.

The locations of the N distinct acoustic scatterers are modeled as random processes and hence we need to simulate the Helmholtz partial differential equation (PDE), for the unknown stochastic scattered field u^s, in a random unbounded region. The PDE is augmented with a radiation condition and mixed boundary conditions are imposed on each of the disjoint boundaries of the obstacles and on the x-axis. The boundary conditions incorporate the impact of the incident wave u^{inc} (and its reflection from the half-plane) on the obstacles. Fig. 1 provides a schematic for an example 20 stochastic-dimensional configuration in the upper half-plane that we use to demonstrate our offline/online reduced basis MC and QMC algorithms.

There is a large literature on the effect of acoustic scattering in the half-plane and also by configurations with multiple particles. See [1] and the extensive list of references therein for various applied mathematics tools for the deterministic

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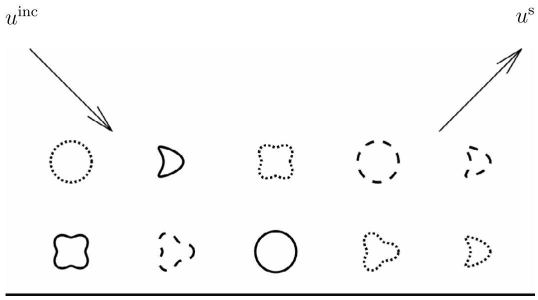

Fig. 1. A 20 stochastic-dimensional model in the upper half-plane: An example configuration in the half-plane, bounded by a sound-soft wall, comprising ten particles in their unperturbed positions. The (x, y) coordinate of each particle is random. The stochastic configuration contains various particle shapes with several material properties. Sound-soft obstacles are indicated with dotted lines $(\cdots)$; impedance boundaries are indicated with dashed lines (- - -); sound-hard obstacles are indicated with solid lines (—).

version of the model problem considered in this article. However, their stochastic configuration multiple particle computational mathematics counterpart, augmenting the half-plane model with high-stochastic-dimensions, has not been explored in the literature.

Uncertainties in configurations of such half-plane model problems arise naturally. Furthermore, optimizing certain QoIs by varying the locations of multiple particles in the half-plane is useful for designing configurations with particular desired scattering effects. Full-space stochastic scattering models, without taking into account half-plane effects, have been recently explored by the authors and other researchers, see [2] and references therein.

In addition to several applications of the half-plane models described in the literature (see [1] and references therein), the half plane problem in this work is motivated by a recent project to apply phase-stepping interferometric microscopy to measure nanowires using light scattering by the nanowires and the surface beneath them [3]. Calibration requires numerical scattering simulation in the half-space bounded by the underlying surface. For long parallel nanowires whose lengths are large compared to their diameters, the electromagnetic field can be computed from the scalar field that satisfies the two dimensional Helmholtz equation in the upper half plane obtained by taking a cross section of the half-space. For the deterministic model in [3], the cross sections of the nanowires are assumed to be circles and simple methodology can be used to simulate the deterministic model.

The quality of the results produced by such simulations depends on knowledge of the exact positions of the nanowires. Numerical evidence, even for simple shapes, shows that simulation results may be inaccurate for uncertain configurations because the model is sensitive to perturbations. A standard technique for such problems is to develop a stochastic PDE model that incorporates the uncertainty in the positions of the nanowires.

In the last decade there have been significant advances in the solution of various stochastic PDE problems using collocation type methods such as Monte Carlo (MC), quasi-Monte Carlo (QMC), stochastic collocation generalized polynomial chaos (gPC), and reduced basis methods (RBM) and their quasi-, multifidelity, sparse-grid and multilevel counterparts [4–12]. In these methods the expected value of the QoI (or other statistical moments) is evaluated using cubature approximation of a multi-dimensional integral with respect to the stochastic variables. For each cubature point (fixed values of the stochastic variables) a simulation of the (deterministic) PDE is required.

The gPC and RBM methods construct a functional approximation of the QoI that replaces simulation of the full PDE model in the cubature. In the gPC method there are several ways to construct the functional approximation, including interpolation [13,14], regression [10], compressive sampling [10,11] and discrete orthogonal projection [15]. If the QoI to be approximated is smooth in the stochastic variables, the gPC or RBM approximations are more efficient than MC. Sparse-grid versions of the gPC do allow increasing the stochastic dimension to a certain degree, with the trade-off being reduced order convergence [7,12]. For a review we refer to [4,16,7] and the book [15]. Certain quasi-MC methods also exhibit high order convergence [16].

The RBM approximates the QoI in a low dimensional space spanned by solutions of the PDE for fixed "snapshot" values of the stochastic variables. The snapshot values are chosen using the greedy algorithm or a weighted greedy algorithm. The efficiency of this method has been demonstrated for various deterministic and stochastic PDEs [13,17,4,14,18]. The construction of the RBM snapshots and associated basis functions (which includes simulation of the full PDE model), is performed offline. The most expensive part in such offline computation is the construction of the basis functions for offline setup of a reduced order model (ROM). For a chosen ROM quality tolerance parameter, a major theoretical problem associated with such a greedy algorithm based offline construction is the development of stopping criteria based on *a posteriori* robust error estimates for the expensive greedy search and setting up the reduced basis functions. Subsequent online simulation of the PDE model at the stochastic cubature points is efficiently performed using the approximation spanned by the reduced bases.

In this work we focus on developing a different kind of offline/online method for the half-plane scattering model. Our method takes advantage of the existing offline/online T-matrix framework for scattering by single particles and the framework does not require construction of basis functions or greedy search of snapshot parameters. The several decade old open problem on the theoretical problem of stopping criteria for construction of the T-matrix based ROM was solved by