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Parametric representation of cell boundary flux distribution in well equations

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ABSTRACT

Material transport is anticipated between adjacent porous media in capillary contact for which we have independent Neumann function solutions to either Poisson's Equation or the Heat Equation. These solutions can be extended by opening the boundary using Green's Theorem, resulting in analytic solutions coupled through a boundary integral. Previously, a parametric representation of the boundary flux was proposed as a linear combination of uniform flux and uniform pressure constituents, which has advantages of analytic evaluation of contributing terms. This boundary flux structure is shown to be exact for cells of identical size and permeability. We highlight the extension to systems of either differing domain size or permeability using prolongation. Prolonged problems allow identification with symmetry of equal cell size problems and an exact solution for flux distribution. Correcting for prolongation requires additional uniform flux and circulation elements that are related to the degree of mismatch in cells in the originally-posed problem. Since lengths are scaled with respect to transport properties, we can claim the new method also allows significant bandwidth reduction in solving the heat equation for heterogeneous systems using parametric representation of flux in boundary integrals.

1. Introduction

Transport problems with spatially varying coefficients are commonplace. While the constant coefficient problem can now be solved analytically for a wide range of geometries, boundary conditions, and source terms, solutions for spatially varying transport problems often require numerical approaches that inherently carry a new set of issues regarding convergence and appropriate discretization [1,2]. Simulations involving transport are currently being run with billions of cells [3], each of which must have specified input for transport properties and initial values with only a sparse set of known values. In such approaches, grids must frequently be refined for accuracy purposes even where transport coefficients are constant [4]. In particular, refinement is required where nonlinear flow is anticipated, such as around wells, faults, and fractures. Wells are of particular interest, since this is where pressure disturbances associated with injection or withdrawal are introduced, and this is often the only source of information gathering from the system. In the case of reservoir engineering simulations, global refinement at the size of a wellbore, nominally with a six inch radius, is prohibitively expensive; thus, local grid refinement has emerged as a necessity in numerical simulations. Machine size and simulation time often dictate the maximum level of discretization that can reasonably entertained.

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On the other hand, computationally efficient analytic solutions to the Heat Equation now exist to model time dependent behavior of reservoirs in the presence of line source functions of arbitrary complexity [5–7]. As these solutions allow tremendous flexibility, yield infinite scalability, require no time marching, and necessitate no subgridding for accuracy, they hold great promise as "infinitely refined grid" solutions. Such solutions, if embedded in a numerical routine, would allow accurate capture of the impact of nonlinear flow fields below the resolution of the numerical grid. These solutions can increase accuracy in purely numerical approaches [8] and higher order stencils associated with the multipoint flux approximation [9–11]. With embedded analytical content, numerical procedures could benefit with the avoidance of local grid refinement without sacrificing accuracy with only the overhead of a coarse grid simulation.

Blending of analytical and numerical solutions can be accomplished by relating analytic results to numerical procedure cell properties. While not analytic, Peaceman [12,13] used high order gridding results to develop an equation to relate coarse cell properties with what one would expect to observe at a well. The use of such well equations is subject to the case restrictions of the fine grid simulations.

Boundary Element/Integral Method has found its elegance in application of fluid flow problems, in which such integral equation techniques widen the range of problems that can be solved [14–16]. Green's Theorem extends a closed system semi-analytic solution to open systems, which allows the role of boundary flux distribution, particularly on well cells, to be examined. Discretization is then relegated to the surface rather than the volume. Consequently, it is then a free boundary problem in that the solution must also prescribe boundary values as opposed to specification of boundary conditions [17]. It is shown that the boundary flux distribution has a significant impact on computed well pressure with increasing significance for decentralized wells and oriented flows [18]. Solutions to coupled Neumann function solutions using Gaussian quadrature to numerically evaluate the boundary integral were posed for heterogeneous transport problems [15,19]. At each boundary node, pressure and flux matching conditions were enforced in terms of the unknown flux magnitude. Required boundary node density for this nonparametric method to incorporate boundary flux increased problem bandwidth significantly, especially when a source or sink function was in close proximity to a surface. Examination of such flux profiles revealed an integrand with spike-like character, requiring splitting of the integral and more boundary nodes. Upon examination of a large number of flux patterns, Hazlett and Babu [18] introduced a parametric representation of boundary flux that adapts to well position and trajectory to solve for coupling between analytic solutions for pressure. For equal cell size problems, only two unknowns per interface need to be introduced with arbitrary choice of two surface locations for pressure matching. In simple two cell time-independent problems, only a single unknown is necessary, and the proposed parametric representation is exact. Such problems are for illustrative purposes and lend themselves to generalization. The current methodology is limited to cell shapes for which there exists a closed-form, computable solution. This currently comprises rectangles, circles, circular sectors, and special case triangles in 2D [6] and their 3D prismatic counterparts. Arbitrary bounded elements can be made using an ensemble of shapes which lend themselves to closed-form solutions to approximate any target geometry [19].

This paper extends the solution of equal-sized regions to those of unequal size. Since lengths are scaled with respect to transport properties, heterogeneous transport property problems can be recast as homogeneous problems of unequal cell size using prolongation [6,20]. This new adaptive parametric representation scheme can tremendously reduce the bandwidth in solving heterogeneous transport problems, needing only two unknowns per interface regardless of problem dimensionality.

2. Development

2.1. Proposed parametric representation

The rectangular domain problem, in the context of capturing below resolution flow fields using analytic solutions, is relevant to practical transport problems. The configuration of cells does not equal to a numerical simulation cell. As an analytic solution, it rather represents an infinitely refined grid. The governing equation represented in the Cartesian system in this work has the form,

$$k_x \frac{\partial^2 \mathrm{P}}{\partial \mathrm{x}^2} + k_y \frac{\partial^2 \mathrm{P}}{\partial \mathrm{y}^2} = \phi \mu C_t - \delta(\mathrm{x} - \mathrm{x_o}) \cdot \delta(\mathrm{y} - \mathrm{y_o}), \tag{1}$$

where k_x and k_y represent directional dependent transport properties, ϕ is a rock property, μ is a fluid viscosity, and C_t stands for the total system compressibility. The second term on the RHS is the point source term represented by a product of Dirac deltas. The above equation can be cast in dimensionless form through the introduction of dimensionless groups and scaled variables,

$$P = \frac{abh\Delta \mathrm{P}}{qB\mu}, \qquad x = \frac{\mathrm{x}}{\sqrt{k_x}}, \qquad y = \frac{\mathrm{y}}{\sqrt{k_y}}$$

where h is the height in the third dimension, and the pressure difference is the difference between local pressure and the volume averaged quantity. A solution with a sealed boundary corresponds to a Neumann function. There are numerous analytic and numerical methods for computing Neumann function [21–23]. The dimensionless Neumann function