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Some results on the upper bound of optimal values in interval convex quadratic programming

Wei Li [a], Mengxue Xia [a], Haohao Li [b,*]

[a] *Institute of Operational Research & Cybernetics, Hangzhou Dianzi University, Hangzhou, 310018, China*

[b] *School of Mathematics and Statistics, Zhejiang University of Finance and Economics, Hangzhou, 310018, China*

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A B S T R A C T

One of the fundamental problems in interval quadratic programming is to compute the range of optimal values. For minimized problem with equality constraint, computing the upper bound of the optimal values is known to be NP-hard. One kind of the effective methods for computing the upper bound of interval quadratic programming is so called dual method, based on the dual property of the problem. To obtain the exact upper bound, the dual methods require that the duality gap is zero. However, it is not an easy task to check whether this condition is true when the data may vary inside intervals. In this paper, we first present an easy and efficient method for checking the zero duality gap. Then some relations between the exact upper bound and the optimal value of the dual model considered in dual methods are discussed in detail. We also report some numerical results and remarks to give an insight into the dual method's behavior.

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1. Introduction

The interval systems and interval mathematical programming (IvMP)[1] have been studied by many authors, see, e.g., [1–15] and a survey paper [16], since intervals naturally appear in many situations when handle inexact data. One of frequent problems in IvMP is to compute the range of optimal values [17,4,18–23]. Some authors studied the problem of computing the range of optimal values of interval quadratic programs (IvQP) [18,19,21,24]. It is known that finding the upper bound of the optimal values in IvQP is a computationally hard problem when the constraint includes interval linear equalities.

There have been developed diverse methods for computing the range of the optimal values in IvQP. Liu [18] and Li [19] described some methods to compute the lower and upper bounds of IvQP with inequality and nonnegative constraints. Hladík [21] studied the more general IvQP and proposed an effective method to compute the bounds of IvQP with both equality and inequality constraints. For computing the upper bound, these methods described in [18,19,21] are based on the dual problem of IvQP (dual method for short), under the condition that the zero duality gap of a pair of primal and dual IvQP is specified. Recently, Li et al. [24] proposed a new method to compute the upper bound of optimal values of IvQP. In this method, only primal program is taken into consideration (primal method for short). The dual problem is not required and thus the condition that the duality gap is zero is also removed.

Just like the primal and dual simplex algorithm for linear programming, both the primal method and the dual method for computing the upper bound of IvQP are very useful and they are suitable for different situations respectively, depending

* Corresponding author.
E-mail address: hlzufe@126.com (H. Li).

[1] We use abbreviation IvMP, instead of IMP, for interval mathematical programming, to avoid confusion with the abbreviation for integer mathematical programming.

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on the structure of IvQP. To obtain the exact upper bound $\bar{f}$ of IvQP, the dual methods first compute the optimal value ψ of the Dorn dual, then one has $\bar{f} = \psi$ if the zero duality gap is assured, otherwise one can only obtain $\bar{f} \geq \psi$.

When using dual method, the difficulty is that it is not an easy task to check whether there is a zero duality gap. In this paper, we first present an easy and efficient method for checking the zero duality gap. Then some relations between $\bar{f}$ and ψ in dual methods are discussed in detail to give an insight into the dual method's behavior.

2. Preliminaries

Following notations from [8], an interval matrix is defined as

$$\mathbf{A} = [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{m\times n} | \underline{A} \leq A \leq \overline{A}\},$$

where $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, $\underline{A} \leq \overline{A}$, and "$\leq$" is understood componentwise. By

$$A_c = \frac{1}{2}(\underline{A} + \overline{A}), A_\Delta = \frac{1}{2}(\overline{A} - \underline{A}),$$

we denote the center and the radius of A, respectively. Then $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$. An interval vector $\mathbf{b} = [\underline{b}, \overline{b}] = \{b \in \mathbb{R}^m | \underline{b} \leq b \leq \overline{b}\}$ is understood as one-column interval matrix.

Let $\{\pm 1\}^m$ be the set of all $\{-1, 1\}$ m-dimensional vectors, i.e.

$$\{\pm 1\}^m = \{y \in \mathbb{R}^m | \mid y \mid = e\},$$

where $e = (1, \dots, 1)^T$ is the m-dimensional vector of all 1's and the absolute value of a matrix $A = (a_{ij})$ is defined by $|A| = (|a_{ij}|)$. For a given $y \in \{\pm 1\}^m$, let

$$T_y = diag(y_1, \dots, y_m)$$

denote the corresponding diagonal matrix. For each $x \in \mathbb{R}^n$, we define its sign vector sgn x by

$$(\text{sgn } x)_i = \begin{cases} 1 & \text{if } x_i \geq 0, \\ -1 & \text{if } x_i < 0, \end{cases}$$

where $i = 1, \dots, n$. Then we have $|x| = T_z x$, where $z = \text{sgn } x \in \{\pm 1\}^n$.

Given an interval matrix $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$, for each $y \in \{\pm 1\}^m$ and $z \in \{\pm 1\}^n$, we define matrices

$$A_{yz} = A_c - T_y A_\Delta T_z.$$

Similarly, for an interval vector $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$ and for each $y \in \{\pm 1\}^m$, we define vectors

$$b_y = b_c + T_y b_\Delta.$$

Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{k\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $d \in \mathbb{R}^k$ and $Q \in \mathbb{R}^{n\times n}$ be given, consider the quadratic programming problem

$$\min \frac{1}{2}x^T Qx + c^T x \quad \text{subject to } Ax \leq b, Bx = d, x \geq 0,$$

where Q is positive semidefinite. Briefly, we rewrite the problem as

$$\text{Min} \left\{ \frac{1}{2}x^T Qx + c^T x | Ax \leq b, Bx = d, x \geq 0 \right\}. \tag{1}$$

The Dorn dual problem [25,26] of the quadratic program (1) is

$$\text{Max} \left\{ -\frac{1}{2}u^T Qu - b^T v - d^T w | Qu + A^T v + B^T w + c \geq 0, v \geq 0 \right\}. \tag{2}$$

Let

$$f(A, B, b, c, d, Q) = \inf \left\{ \frac{1}{2}x^T Qx + c^T x | Ax \leq b, Bx = d, x \geq 0 \right\}$$

and

$$g(A, B, b, c, d, Q) = \sup \left\{ -\frac{1}{2}u^T Qu - b^T v - d^T w | Qu + A^T v + B^T w + c \geq 0, v \geq 0 \right\} \tag{3}$$

denote the optimal values of (1) and (2), respectively.

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