Journal of Computational and Applied Mathematics 302 (2016) 224–233

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A hybrid optimization method for multiplicative noise and blur removal

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ARTICLE INFO

Article history:
Received 15 November 2014
Received in revised form 19 December 2015

Keywords:
Image restoration
Multiplicative noise
Quadratic penalty function method
Alternating direction method

ABSTRACT

The main contribution of this paper is to propose a new hybrid optimization method for the multiplicative noise and blur removal problem. A degraded image can often be recovered efficiently by minimizing an objective function which consists of a data-fidelity term and a regularization term. In the paper, we apply the quadratic penalty function method combined with the alternating direction method to minimize the corresponding objective function. Numerical experiments are presented to demonstrate the effectiveness of the proposed method. Experimental results illustrate the state-of-the-art performance of the proposed method to handle multiplicative noise and blur removal problem.

1. Introduction

Images are usually degraded by blur and noise during images acquisition and transmission. Image restoration is a widely studied problem in the applied mathematics community. Most of the relevant literatures have been devoted to deal with the additive noise model, and they usually assume that the degraded image $\mathbf{f} \in \mathbb{R}^n$ is given by

$$\mathbf{f} = \mathbf{H}\mathbf{u} + \mathbf{n},$$

where the original image $\mathbf{u} \in \mathbb{R}^n$ is corrupted by a spatial-invariant blur matrix $\mathbf{H} \in \mathbb{R}^{n\times n}$ and an additive gaussian noise $\mathbf{n} \in \mathbb{R}^n$. There are various approaches to reconstruct image under this additive noise scheme, we refer the reader to [1–3] and references therein for a view of this subject.

In recent years, many researchers have paid attention to other kinds of random noise, such as multiplicative noise [4–9], impulse noise [10–14] and Poisson noise [15]. In this paper, we focus on the deblurring issue under the multiplicative noise. Assume that the original image $\mathbf{u}$ is blurred by a spatially invariant blur $\mathbf{H}$ and corrupted by a multiplicative noise $\mathbf{n}$, i.e.,

$$\mathbf{f} = \mathbf{H}\mathbf{u} \circ \mathbf{n}, \tag{1.1}$$

where $\circ$ refers to the componentwise multiplication. Unlike the additive noise model, the presence of multiplicative noise destroys the image in a totally different way, and the image reconstruction problem under the multiplicative noise is evidently challenging. Multiplicative noise removal is very important and has many applications [4,7], for example, speckle noise in synthetic aperture radar (SAR) images, speckle noise in ultrasound, and magnetic field inhomogeneity in magnetic resonance imaging (MRI). In the paper, $\mathbf{n}$ is assumed to follow the Gamma distribution.

For the simplest case when $\mathbf{H} = \mathbf{I}$ is an identity matrix, several approaches have been proposed to handle this multiplicative noise removal problems, such as variational approaches [4,5,16,6,17,8], filtering approaches [18,19]. Among the variational approaches, total variation (TV) based models seem to be quite interesting and efficient since the TV

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http://dx.doi.org/10.1016/j.cam.2016.02.013
0377-0427/

formulation has the ability to preserve edges [4,7,20]. Recently, Aubert and Aujol [4] used a maximum a posteriori (MAP) estimator and derived a functional whose minimizer corresponds to the denoised image to be recovered in the multiplicative noise removal problem. Since the model in [4] is not convex, Huang et al. [6] took logarithms on both sides of $\mathbf{f} = \mathbf{un}$, i.e., $\log \mathbf{f} = \log \mathbf{u} + \log \mathbf{n}$, then proposed a convex model which is strongly related to AA model and obtains a better result. Note that the idea of converting the multiplicative model into an additive one by taking logarithms is usually considered in literatures, such as [5,17,9]. In [16], the multiplicative noise removal problem is handled by using $L1$ date-fidelity term on frame coefficients and their hybrid approach has got quite well results.

On the other hand, there are some papers dealing with the blur and multiplicative noise removal problem, see [4,21,6,22,7,9,23]. In [7], Rudin, Lions and Osher considered to minimize the total variation of $\mathbf{u}$ under two constraints (RLO model)

$$\begin{aligned} \min_{\mathbf{u}\in\mathbb{R}^n} \quad & TV(\mathbf{u}) \\ \text{s.t.} \quad & \int \frac{\mathbf{f}}{\mathbf{Hu}} = 1, \qquad \int \left(\frac{\mathbf{f}}{\mathbf{Hu}} - 1\right)^2 = \sigma^2, \end{aligned} \tag{1.2}$$

where the mean of noise is assumed to be 1, and the variance of noise is equal to σ^2. In [4], Aubert and Aujol had also extended their model to handle the deblurring problem (AA model)

$$\min_{\mathbf{u}\in\mathbb{R}^n} \int \left(\log(\mathbf{Hu}) + \frac{\mathbf{f}}{\mathbf{Hu}}\right) + \lambda TV(\mathbf{u}). \tag{1.3}$$

The objective functions of RLO model and AA model are both non-convex and it is very expensive to find the minimizers by using the gradient projection method. In [6], Huang, Ng, and Wen converted the non-convex model to a convex model through a logarithmic transformation, and encountered some difficulties when they used the same way to handle the deblurring problem. In [9], Wang and Ng proposed a variational approach composed of a L_1 data-fidelity term and a TV regularization term in the log-image domain. They approximated the non-convex constraints by a set of convex constraints, and applied the alternating direction method to solve the resulting optimization problem. In [22], Huang, Ng, and Zeng considered a general model $\mathbf{f} = (\mathbf{Hu} + n_1) \circ \mathbf{n}_2$, where $\mathbf{n}_1$ is a weak Gaussian noise and $\mathbf{n}_2$ is a multiplicative noise. Based on the model, they derive a new variational model and use the convex relaxation technique to ensure the energy function is convex. In addition, they also consider the case that the additive Gaussian noise $\mathbf{n}_1$ does not exist, which is equivalent to the model (1.1). Recently, Huang, Lu, and Zeng [21] proposed a two-step approach to handle the multiplicative noise and blur removal problem. The approach reduced the multiplicative noise by nonlocal filters, and then employed a convex variational model to obtain the final restored images where the model composed of an L_1-L_2 data-fidelity term and a total variation regularization term. In [23], Zhao, Wang, and Ng proposed a new convex optimization model for the multiplicative noise and blur removal problem by rewriting the multiplicative noise and blur equation to decouple the image variable and the noise variable. Their method can handle blur and multiplicative noise removal quite well.

The main contribution of this paper is to propose a new hybrid optimization method to handle the multiplicative noise and blur removal problem. The corresponding objective function is minimized by employing the quadratic penalty function method combined with the alternating direction method of multipliers. The discrete form of AA model is

$$\min_{\mathbf{u}\in\mathbb{R}^n} \sum_{i=1}^{n} \left(\log[\mathbf{Hu}]_i + \frac{[\mathbf{f}]_i}{[\mathbf{Hu}]_i}\right) + \lambda \|\mathbf{u}\|_{TV}. \tag{1.4}$$

Since the data-fidelity term in (1.4) is not convex, we cannot solve the non-convex optimization problem simply by using the augmented Lagrangian method and the alternating direction method. Instead, in this paper, we apply the quadratic penalty function method together with the alternating direction method to solve the problem (1.4).

The outline of this paper is as follow. In Section 2, we study the quadratic penalty function method combined with the alternating direction method to solve the separate non-convex constrained optimization problem. Then we apply the hybrid optimization approach discussed in Section 2 to handle the multiplicative noise and blur removal problem in Section 3. In Section 4, experimental results are reported to demonstrate the effectiveness of the proposed method. Finally, some concluding remarks are given in Section 5.

2. Hybrid optimization method

In this section, we describe the new hybrid optimization method. Firstly, consider an unconstrained optimization problem which has the following form

$$\min_{\mathbf{u}\in\mathbb{R}^n} f_1(\mathbf{u}) + f_2(g(\mathbf{u})). \tag{2.5}$$

Sometimes, it is difficult to solve the unconstrained problem, then we can apply the variable splitting method to create an equivalent constrained problem which is easier to solve. The idea is to create a new variable $\mathbf{w}$ to serve as the argument of f_2, under the constraint $\mathbf{w} = g(\mathbf{u})$. The equivalent constrained problem to (2.5) is

$$\begin{aligned} \min_{\mathbf{u},\mathbf{w}\in\mathbb{R}^n} \quad & f_1(\mathbf{u}) + f_2(\mathbf{w}) \\ \text{s.t.} \quad & \mathbf{w} = g(\mathbf{u}). \end{aligned} \tag{2.6}$$