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Two-sweep modulus-based matrix splitting iteration methods for linear complementarity problems[☆]

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ABSTRACT

In this paper, we will extend the two-sweep iteration methods to solve the linear complementarity problems and establish a class of two-sweep modulus-based matrix splitting iteration methods for the implicit fixed-point equation of the linear complementarity problems. Some convergence properties of two-sweep modulus-based matrix splitting iteration methods are discussed when the system matrices are positive-definite matrices and H_+-matrices. Numerical experiments are presented to illustrate the efficiency of the proposed methods.

1. Introduction

For a given matrix $A \in \mathbb{R}^{n\times n}$ and vector $q \in \mathbb{R}^n$, the linear complementarity problems (LCP(q, A)) is that one tries to find $w \in \mathbb{R}^n$ and $z \in \mathbb{R}^n$ satisfy

$$w := Az + q \geq 0, \quad z \geq 0 \quad \text{and} \quad z^T w = 0. \tag{1.1}$$

LCP(q, A) of the form (1.1) often appears in many different applications of scientific computing, such as the linear and quadratic programming, the economies with institutional restrictions upon prices, the optimal stopping in Markov chain and the free boundary problems. One can see [1–4] for more details.

Based on simplex type processes, the pivot algorithms are established in [5] to obtain the numerical solution of the LCP(q, A) and may not be popular because it may destroy sparsity and require too many pivots in actual implementations [5]. To improve the computational techniques for the numerical solution of the LCP(q, A), some iteration methods have been proposed, such as the projected successive overrelaxation (SOR) iteration methods [6], the general fixed-point iteration methods [7–10].

Based on transforming the LCP(q, A) into a linear system in a fixed-point form, when A is symmetric positive definite, a modulus iteration method with polynomial complexity was proposed [11], which is defined as the solution of a system of

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linear equations at each iteration, see also Section 9.2 in [3]. In [5], the modulus iteration method was extended to a class of nonsymmetric matrix. Combining the modulus iteration method with an auxiliary parameter, a modified modulus iteration method was proposed in [12], which is suitable for the symmetric positive definite A. Whereafter, based on the matrix splitting of the system matrix A, a class of modulus-based matrix splitting iteration methods was presented in [13], which can be considered as a general framework for solving the LCP(q, A). The advantage of the modulus-based matrix splitting iteration methods is that it not only covers some presented iteration methods, such as the modified modulus method [12] and the nonstationary extrapolated modulus methods [14], but also yields a series of modulus-based relaxation methods, such as modulus-based Jacobi, Gauss–Seidel, SOR and accelerated over relaxation (AOR) iteration methods. The convergence condition of modulus-based matrix splitting iteration methods from the condition of a compatible H-splitting to that of an H-splitting was discussed in [15].

According to the promising behaviors and elegant mathematical properties of the modulus-based matrix splitting iterative scheme, it immediately attracted considerable attention, resulting in numerous papers devoted to various aspects of the new algorithm, such as modulus-based synchronous multisplitting iteration methods [16], modulus-based synchronous two-stage multisplitting iteration methods [17], accelerated modulus-based matrix splitting methods [18,19], two-step modulus-based matrix splitting iteration methods [20], general modulus-based matrix splitting methods [21], scaled extrapolated block modulus algorithms [22] and nonstationary extrapolated modulus algorithms [14].

In this paper, we will extend the two-sweep iteration methods [23–27] to solve the linear complementarity problems. Our approach is to introduce an identical equation [24,25] to establish a class of two-sweep modulus-based matrix splitting iteration methods for solving the LCP(q, A). Based on the different parameter matrices, the proposed methods can yield a series of two-sweep modulus-based relaxation methods. The convergence of the proposed methods is discussed when the system matrix is either a positive-definite matrix or an H_+-matrix. Numerical experiments are to illustrate the efficiency of the proposed methods.

This paper is organized as follows. Some necessary definitions and lemmas are reviewed in Section 2. The two-sweep modulus-based matrix splitting iteration methods are established in Section 3. The convergence conditions when the system matrix is either a positive-definite matrix or an H_+-matrix are presented in Section 4. Numerical experiments are reported in Section 5, and finally some concluding remarks are given in Section 6.

2. Preliminaries

Some necessary definitions, notations and lemmas are reviewed in this section, which are used in the sequel discussions.

A matrix $A \in \mathbb{R}^{n\times n}$ is called a P-matrix if all of its principle minors are positive [28]. It follows that a matrix A is a P-matrix if and only if the LCP(q, A) has a unique solution for all $q \in \mathbb{R}^n$ [4]. A sufficient condition for the matrix A to be a P-matrix is that A is a positive-definite matrix or an H-matrix with positive diagonals (H_+-matrix [29,30]). $|A|$ denotes the nonnegative matrix with entries $|a_{ij}|$, A^T denotes the transpose of the matrix A and $\rho(A)$ denotes the spectral radius of the matrix A. A is an H-matrix if its comparison matrix $\langle A\rangle = (\langle a\rangle_{ij}) \in \mathbb{R}^{n\times n}$ is an M-matrix, where

$$\langle a\rangle_{ij} = \begin{cases} |a_{ij}| & \text{for } i = j, \\ -|a_{ij}| & \text{for } i \neq j, \end{cases} \qquad i, j = 1, 2, \ldots, n.$$

A matrix $A \in \mathbb{R}^{n\times n}$ is called a Z-matrix if its off-diagonal entries are non-positive; an M-matrix if A is a Z-matrix and $A^{-1} \geq 0$. If A is an M-matrix and B is a Z-matrix, then $A \leq B$ implies that B is an M-matrix [31].

Lemma 2.1 ([32]). *Let $A \in \mathbb{R}^{n\times n}$ and suppose that there exists $u \in \mathbb{R}^n$, with $u > 0$ such that $|A|u < u$. Then there exists a constant $\theta \in [0, 1)$ such that $\rho(A) \leq \theta$.*

Lemma 2.2 ([33]). *Let $A \in \mathbb{R}^{n\times n}$ be an H-matrix and $A = D - B$, where D is the diagonal part of the matrix A. Then the following statements hold true:*

(i) *A is nonsingular and $|A^{-1}| \leq \langle A\rangle^{-1}$;*
(ii) *$|D|$ is nonsingular and $\rho(|D|^{-1}|B|) < 1$.*

Lemma 2.3 ([26]). *Let*

$$A = \begin{bmatrix} B & C \\ I & 0 \end{bmatrix} \geq 0 \quad \text{and} \quad \rho(B + C) < 1.$$

Then $\rho(A) < 1$.

3. Two-sweep modulus-based matrix splitting iteration methods

To obtain the two-sweep modulus-based matrix splitting iteration methods, a brief review of the classical modulus-based matrix splitting iteration methods is required.