Journal of Computational and Applied Mathematics 301 (2016) 44–52

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

On numerical homogenization of shale gas transport

Alexey Talonov [a], Maria Vasilyeva [b,c,*]

[a] Department of Physics, National University of Science and Technology "MISiS", Leninsky Pr. 4, Moscow, 119991, Russia

[b] Department of Computational Technologies, Institute of Mathematics and Informatics, North-Eastern Federal University, Yakutsk, Republic of Sakha (Yakutia), 677980, Russia

[c] Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, 630090, Russia

ARTICLE INFO

Article history:
Received 27 November 2015
Received in revised form 14 December 2015

Keywords:
Numerical homogenization
Shale gas transport
Organic rich shale
Nonlinear parabolic equation
Effective properties

ABSTRACT

In this paper, we study the numerical homogenization for the gas transport in organic rich shale with heterogeneous kerogen distribution. We consider organic rich shale as the domain with two subdomains: inorganic matrix and kerogen. The processes in both regions are described by nonlinear parabolic equations, which take into account the filtration, diffusion, and adsorption. We follow the work of Yucel Akkutlu et al. (2015), where the authors develop homogenization techniques for the gas transport in organic rich shale. We use the framework of Yucel Akkutlu et al. (2015) and develop numerical homogenization for two dimensional examples. We discuss local subgrid calculations and the approaches to compute the effective properties numerically. In our methods, the local problems use Dirichlet boundary conditions. We compare the properties of the fine-grid reference solution against those of the coarse grid model. Our approaches show a good agreement between macroscopic quantities.

1. Introduction

Shale gas usually has very low permeability, and consists of organic matrix and kerogen. Kerogen is considered as an organic material that is dispersed in inorganic matrix. The kerogen brings in new fluid storage and transport qualities to the shale [1–3]. Heterogeneities are associated with the nonuniform kerogen distribution in space. This kerogen distribution is complex and usually cannot be represented with periodic representative volumes. For this reason, one may need to use very large representative volumes for upscaling purposes and rely on numerical homogenization techniques. This is investigated in the paper.

The mathematical model of the gas transport in organic rich shales follows the model formulated by Akkutlu and Fathi (2012), [4]. In work [4], the gas transport in organic rich shales involves a matrix with dual porosity continua associated with organic and inorganic pores. In work [5], the authors derive a homogenized model in the form of generalized nonlinear diffusion model to describe the effects of kerogen. In our previous paper [6], we developed a multiscale model reduction technique that describes the shale gas transport in fractured media, where the interaction between the matrix and the fractures is modeled via multiscale basis functions [7–10]. The multiscale approaches developed in [6], where a set of macroscopic models are proposed that take into account the nanoporous nature and nonlinear processes of the shale matrix. In this work, the case of the periodic spatial heterogeneities is considered and a formal asymptotic analysis is

* Corresponding author at: Department of Computational Technologies, Institute of Mathematics and Informatics, North-Eastern Federal University, Yakutsk, Republic of Sakha (Yakutia), 677980, Russia.
E-mail address: vasilyevadotmdotv@gmail.com (M. Vasilyeva).

http://dx.doi.org/10.1016/j.cam.2016.01.021
0377-0427/

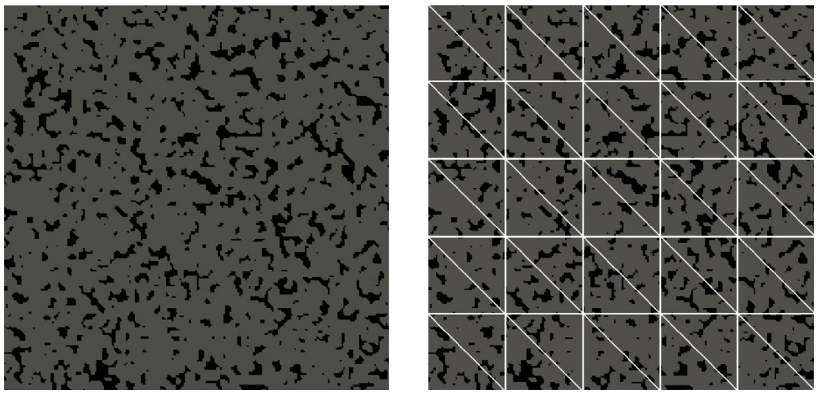

Fig. 1. Two dimensional domain of an organic rich shale with randomly dispersed kerogen in an inorganic matrix (left) and coarse blocks for calculation of the macroscopic upscaled parameters (right). Black: kerogen. Gray: inorganic matrix.

applied to derive macroscopic equations. In the case of non-periodic distribution of matrix heterogeneities, numerical upscaling techniques are needed for calculation of the macroscopic coefficients. In this paper, we propose this numerical homogenization and test the approach on some representative heterogeneities.

In this work, we consider organic rich shale as a domain with two subdomains: (1) inorganic matrix and (2) kerogen. These subdomains can be represented on the fine grid. The kerogen subdomain is characterized by high porosity and very low permeability; the subdomain with inorganic matrix has low porosity and low permeability. The processes in both regions are described by nonlinear parabolic equations, which take into account the filtration, diffusion, and adsorption. In this paper, we develop numerical homogenization technique following the homogenization developed in [5]. In the proposed numerical homogenization (see, [11–14] for general numerical homogenization), the macroscopic coefficients are computed by using the local problems for a given concentration level. Using the macroscopic tensor coefficients, we solve the global coarse-grid problem. We present numerical results and compare with the reference fine-scale solution. For comparison, we use the solution in the x_1-direction in the center of domain for different time steps and the average value of the solution in computational domain. Our numerical results show a good agreement and show that numerical homogenization techniques can be effectively used for representing matrix heterogeneities.

The paper is organized as follows. In Section 2, we present the problem formulation. The fine-scale discretization is presented in Section 3. In Section 4, we present the numerical homogenization approach. Finally, the numerical results are presented in Section 5.

2. Problem formulation

We consider the model of the gas transport in organic rich shales. The shale domain is considered as domain with two subdomains

$$\Omega = \Omega_i \cup \Omega_k,$$

where Ω_k and Ω_i are the subdomains with kerogen and inorganic matrix, respectively (Fig. 1).

In the model of the gas transport in organic rich shales [4], the free gas in the inorganic matrix is transported by the mechanisms of filtration and molecular diffusion. In the kerogen, we have free and sorbed gas, c_μ. The sorbed gas is transported by the surface diffusion mechanism. For the sorbed gas, the mass balance is given using Langmuir kinetics which can be reduced to the Langmuir isotherm representing adsorption, $c_\mu = F(c)$, where c is the free gas amount. For free gas mass balance in the kerogen, we have following unsteady nonlinear equation

$$\left(\phi_k + (1-\phi_k)F_c'\right)\frac{\partial c}{\partial t} = \nabla\cdot\left((\phi_k D_k + (1-\phi_k)D_s F_c')\nabla c\right), \quad \text{in } \Omega_k, \tag{1}$$

where $c = c(x,t)$ represents the amount of free gas, ϕ_k is kerogen porosity, F_c can be defined by Henry's law (linear) or Langmuir isotherms (nonlinear) and

$$F_c' = \frac{dF}{dc}, \quad F = F(c).$$

For the kerogen, the diffusive molecular transport dominates and D_k is the tortuosity-corrected coefficient of the diffusive molecular transport for the free gas in kerogen and D_s is adsorbed-phase diffusivity. The diffusion coefficients are considered in terms of a total mass flux, which is taken from [4].

For free gas mass balance in the inorganic matrix, we have

$$\phi_i\frac{\partial c}{\partial t} = \nabla\cdot\left(\left(\phi_i D + (1-\phi_i)cRT\frac{k_m}{\mu}\right)\nabla c\right), \quad \text{in } \Omega_i, \tag{2}$$