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Synchronization for time-varying complex networks based on control[☆]

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ABSTRACT

In this paper, the problem on synchronization is investigated for complex networks with time-varying delay and without time-varying delay by using control strategy. Together with some Lyapunov–Krasovskii functions and effective mathematical techniques, several conditions are derived to guarantee a class of complex networks with time-varying and without time-varying delays to be synchronized. Finally, examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

Complex networks lie in many fields of our daily life, such as the internet, world wide web, communication networks, social networks, genetic regulatory networks, power grid networks, and so on. Over the past two decades, complex networks have been intensively investigated in various disciplines, such as mathematics, physics, biology, engineering, and social sciences. Most previous researches are focused on the synchronization of forming a single network, which has linearly coupled term. In this paper, we not only study this problem, but also consider that the system has been influenced by nodes with time delay.

Recent decades witness that synchronization in complex networks has attracted increasing interests from many research and application fields, such as secure communication, image processing, and harmonic oscillation generation. The word 'synchronization' comes from Greek, which means 'share time' and today, it comes to be considered as 'time coherence of different processes'. Recently, one of the interesting and significant phenomena in complex dynamical networks is the synchronization of all dynamical nodes in a network. More recently, synchronization in networks or coupled oscillators has received an increasing attention. Hence, the problem of synchronization of time-delay systems has attracted considerable attention during the past few years. However, in many real situations, some networks cannot achieve synchronization directly. Therefore, some control schemes are adopted to design controllers, such as adaptive control, feedback control, observer-based control, impulsive control and intermittent control.

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Chaos synchronization means making two systems oscillate in a synchronized manner, and it has become a very important goal and a subject of much on-going research due to its important applications in secret communication. For the reason that chaos is very sensitive to its initial condition, chaos control and chaos synchronization were once believed to be impossible until the 1990s when Ott et al. [1] developed the OGY method to suppress chaos. In recent years, owing to wide applications, the problem on synchronization in various dynamical networks has been extensively studied in literature [2–19]. Several synchronous criterion are presented to guarantee the convergence of the pinning process locally and globally by construction of Lyapunov functions [5]. L. Wang and Y. Sun [6] have studied the robustness problem of pinning a general complex dynamical network toward an assigned synchronous evolution. The problem of synchronization for a class of complex delayed dynamical networks via pinning periodically intermittent control has been considered in [8]. Some novel and useful exponential synchronous criterion have been obtained by utilizing the methods which are different from the techniques employed in the existing works. The cluster mixed synchronization of these networks has been studied by using some linear pinning control schemes [13,14,18]. Only the nodes in one community, which have direct connections to the nodes in other communities, have been controlled. Adaptive coupling strength method has been adopted to achieve the synchronization as well [14].

Motivated by the above discussions, in this paper, we will study the synchronization of complex dynamical networks with non-delayed and delayed coupling. Combining the control method with linear matrix inequality technique, some sufficient conditions for the synchronization of the time-varying networks are derived. With the proposed synchronization scheme, through adding the simple linear feedback and adaptive controllers to a fraction of nodes in the response network, we can obtain the synchronization conditions for complex networks with time-varying delay and without time-varying delay, respectively. Finally, examples are given to illustrate the effectiveness of the proposed methods.

This paper is organized as follows. The network model is introduced and some necessary lemmas are given in Section 2. Section 3 discusses the synchronization of the complex dynamical networks with non-delayed and delayed coupling by the control method. Corresponding synchronous criterion for guaranteeing the synchronization are obtained. The theoretical results are verified numerically by several representative examples in Section 4. Finally, this paper is concluded in Section 5.

Notation: Throughout this paper, $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For symmetric matrices X and Y, the notation $X > Y$ $(X \geq Y)$ means that the matrix $X-Y$ is positive definite (nonnegative). $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ with $*$ denoting the symmetric term in a symmetric matrix.

2. Preliminaries

Consider a complex dynamical network consisting of N identical nodes with linear couplings network with time-varying delay:

$$\dot{x}_i(t) = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau(t))) + \sum_{j=1}^{N} G_{ij}Dx_j(t) + \sum_{j=1}^{N} G_{ij}Hx_j(t-\tau(t)) + u_i(t), \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^T \in \Re^n$ is the state vector of node i, $f : \Re^n \to \Re^n$ is continuously differentiable, $f(x_i(t)) = [f_1(x_{i1}(t)), \ldots, f_n(x_{in}(t))]^T$, $C = diag(c_1, c_2, \ldots, c_n) \in \Re^{n\times n}$ is a diagonal matrix with positive diagonal entries $c_i > 0$, $i = 1, 2, \ldots, n$. The matrices $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are weight and delayed weight matrices, respectively. The matrices $D = diag(d_1, d_2, \ldots, d_n) \in \Re^{n\times n}$ and $H = diag(h_1, h_2, \ldots, h_n) \in \Re^{n\times n}$ are constants and delayed inner coupling matrices of complex networks, respectively. $G = (G_{ij})_{N\times N}$ is outer coupling matrix representing the complex network topology, defined as follows: if there is a connection between nodes i and j $(j \neq i)$, then $G_{ij} \neq 0$; otherwise, $G_{ij} = 0$ $(j \neq i)$ and the diagonal elements of matrix G are defined by $G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij}$.

Assumption 1. Here $\tau(t)$ denotes the interval time-varying delay satisfying

$$0 \leq \tau_0 \leq \tau(t) \leq \tau_m, \qquad \mu_0 \leq \dot{\tau}(t) \leq \mu_m < +\infty. \tag{2}$$

Assumption 2.

$$\gamma_i^- \leq \frac{f_i(x) - f_i(y)}{x-y} \leq \gamma_i^+, \quad \forall x, y \in \Re,\ x \neq y,\ i = 1, 2, \ldots, n, \tag{3}$$

where γ_i^-, γ_i^+, $i = 1, 2, \ldots, n$ are constants and $\Gamma_1 = diag(\gamma_1^-, \gamma_2^-, \ldots, \gamma_n^-)$, $\Gamma_2 = diag(\gamma_1^+, \gamma_2^+, \ldots, \gamma_n^+)$.

In this paper, we consider one special case that time-varying delay is available, the controller is designed:

$$u_i(t) = -d(t)(x_i(t) - s(t)) - \sum_{j=1}^{N} G_{ij}Ds(t) - \sum_{j=1}^{N} G_{ij}Hs(t-\tau(t)), \quad i = 1, 2, \ldots, N, \tag{4}$$