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On the superlinear local convergence of a penalty-free method for nonlinear semidefinite programming ☆

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ABSTRACT

This paper is concerned with a sequentially semidefinite programming (SSDP) algorithm for solving nonlinear semidefinite programming problems (NLSDP), which does not use a penalty function or a filter. This method, inspired by the classic SQP method, calculates a trial step by a quadratic semidefinite programming subproblem at each iteration. The trial step is determined such that either the value of the objective function or the measure of constraint violation is sufficiently reduced. In order to guarantee global convergence, the measure of constraint violation in each iteration is required not to exceed a progressively decreasing limit. We prove the global convergence properties of the algorithm under mild assumptions. We also analyze the local behaviour of the proposed method while using a second order correction strategy to avoid Maratos effect. We prove that, under the strict complementarity and the strong second order sufficient conditions with the sigma term, the rate of local convergence is superlinear. Finally, some numerical results with nonlinear semidefinite programming formulation of control design problem with the data contained in *COMPl$_e$ib* are given.

1. Introduction

We consider the following nonlinear semidefinite programming problem (NLSDP)

$$\begin{aligned} \min \quad & f(x), \\ \text{s. t.} \quad & h(x) = 0, \\ & G(x) \preceq 0, \end{aligned} \tag{1.1}$$

where $f : R^n \to R, h : R^n \to R^p, G : R^n \to S^m$ are smooth functions, S^m denotes the set of mth order real symmetric matrices. In this article we use the inner product $\langle A, B\rangle = trace(AB)$ for all matrices $A, B \in S^m$. $\preceq$ denotes the negative semidefinite order (that is $A \preceq BiffA - B$ is a negative semidefinite matrix). $A \in S^m_-$ means $A \preceq 0$. The order relations $\prec, \succeq$ and $\succ$ are defined similarly.

Nonlinear semidefinite programming has recently become a focal point in optimization research, because such problems arise in many application fields, which include modeling in feedback control [1], structural optimization [2], truss design problems [3], robust control [4], and so forth. Basic theoretical issues of NLSDP such as optimality conditions, stability analysis, constraint nondegeneracy and duality theory have been studied, see [5,6] and the references therein. Some

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algorithms for solving NLSDP have been proposed. Mosheyev et al. [7] proposed a penalty/barrier multiplier method for solving convex SDP problems with a linear matrix inequality constraint. Kočvara and Stingl [8] developed a computer code PENNON for solving NLSDP, in which the augmented Lagrangian function method was used. Fares et al. [9] also applied an augmented Lagrangian method to a class of LMI-constraint problems. Yamashita et al. [10] proposed a primal–dual interior point method for NLSDP, which consists of the outer iteration that finds a KKT point and the inner iteration that calculates an approximate barrier KKT point.

Due to the good behaviour of the SQP method for nonlinear programming, some researchers have generalized the SQP method to solve NLSDP. Fares et al. [4] applied sequential semidefinite programming (SSDP) technique for robust control problems. Correa and Ramirez [11] proposed a line search SSDP method for solving NLSDP and analyzed its global convergence properties. These methods solve a quadratic semidefinite programming subproblem at each iteration to generate a search direction and use the l_1 exact penalty function as a merit function, which determines whether to accept the trial step or not.

In this paper, a new line search SSDP algorithm for (1.1) is proposed. Since a SSDP algorithm with l_1 exact penalty function may become less effective when the penalty factor becomes too large, the new algorithm does not use any penalty function. As a result of that defect, some researchers already extended the filter-SQP idea [12] – a kind of penalty-free method – to solve NLSDP. For example, Gómez and Ramirez [13] proposed a filter algorithm while Chen and Miao [14] proposed a penalty-free method with trust-region framework for NLSDP. They all analyzed the global convergence of the proposed algorithm. We borrow ideas from another penalty-free method (see Chen et al. [15] and Ge et al. [16]) for nonlinear programming and propose a new algorithm for NLSDP. This algorithm uses a successively decreasing limit on the measure of constraint violation as a safeguard for feasibility while the trial step is determined such that either the value of the objective function or the measure of the constraint violation is sufficiently reduced. Under mild assumptions, the global convergence of the proposed algorithm is analyzed. Since the problem (1.1) contains a semidefinite constraint, some definitions and deductions are much different comparing to that in [15,16]. The generalization of this penalty-free method is not trivial.

The local properties for SSDP method, such as convergence rate and sensitivity have been studied by some researchers, see [4,11,17–19]. Those analyses are extensions based on some SQP-frame local convergence results. However, to our knowledge, in the papers which discuss local superlinear convergence rate of the SSDP method, a full size step is taken as default [4,17,19]. In the new algorithm, we do not assume that a full size step is taken as default. This may lead to the rejection of a superlinear convergent step, which is known as the Maratos effect in classic SQP method. To overcome this defect, we adapt a second-order correction technique to modify the algorithm and prove that a full size step with this modification can be accepted for k sufficiently large. It should be pointed out that the algorithm uses an approximate matrix for the Hessian matrix of the Lagrangian function, which is also different from [11,19]. Under the strict complementarity and the strong second order sufficient condition with the sigma term, we prove that the sequence generated by the modified algorithm has local superlinear convergence rate.

The paper is organized as follows. In Section 2, we will introduce some notations and preliminaries. In Section 3, we present the algorithm and analyze its global convergence properties. In Section 4 we discuss the local convergence properties of the modified algorithm. In Section 5, the preliminary numerical results are reported. Finally some remarks are given.

2. Notations and preliminaries

Throughout this paper, we define $g(x) = \nabla f(x)$ and $\nabla^2 f(x)$ as the gradient and the Hessian matrix of the objective function $f(x)$, respectively. $Dh(x)$ as the $p \times n$ Jacobian matrix of $h(x)$, i.e.,

$$(Dh(x))^T = (\nabla h_1(x), \nabla h_2(x), \dots, \nabla h_p(x)).$$

A linear operator $DG(x)$ is defined as

$$DG(x) = \left(\frac{\partial G(x)}{\partial x_1}, \frac{\partial G(x)}{\partial x_2}, \dots, \frac{\partial G(x)}{\partial x_n} \right)$$

and

$$DG(x)d := \sum_{i=1}^{n} \frac{\partial G(x)}{\partial x_i} d_i, \quad \forall d \in R^n.$$

The formula for the adjoint operator $DG(x)^T$ is

$$DG(x)^T Z := \left(\left\langle \frac{\partial G(x)}{\partial x_1}, Z \right\rangle, \left\langle \frac{\partial G(x)}{\partial x_2}, Z \right\rangle, \dots, \left\langle \frac{\partial G(x)}{\partial x_n}, Z \right\rangle \right)^T, \quad \forall Z \in S^m.$$

The operator $D^2G(x) : R^n \times R^n \to S^m$, obtained from the second derivative of $G(x)$, is defined by

$$d^T D^2 G(x) \tilde{d} = \sum_{i,j=1}^{n} d_i \tilde{d}_j \frac{\partial^2 G(x)}{\partial x_i \partial x_j}.$$