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Differential geometry of non-transversal intersection curves of three implicit hypersurfaces in Euclidean 4-space

O. Aléssio [a,*], M. Düldül [b], B. Uyar Düldül [c], Nassar H. Abdel-All [d], Sayed Abdel-Naeim Badr [d]

[a] UFTM - Triângulo Mineiro Federal University, Institute of Exact Sciences, Natural and Education, Department of Mathematics, Uberaba, MG, Brazil

[b] Yıldız Technical University, Science and Arts Faculty, Department of Mathematics, İstanbul, Turkey

[c] Yıldız Technical University, Education Faculty, Department of Mathematics Education, İstanbul, Turkey

[d] Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt

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ABSTRACT

The aim of this paper is to compute all the Frenet apparatus of non-transversal intersection curves (hyper-curves) of three implicit hypersurfaces in Euclidean 4-space. The tangential direction at a transversal intersection point can be computed easily, but at a non-transversal intersection point, it is difficult to calculate even the tangent vector. If three normal vectors are parallel at a point, the intersection is "tangential intersection"; and if three normal vectors are not parallel but are linearly dependent at a point, we have "almost tangential" intersection at the intersection point. We give algorithms for each case to find the Frenet vectors ($\mathbf{t}, \mathbf{n}, \mathbf{b_1}, \mathbf{b_2}$) and the curvatures ($k_1, k_2, k_3$) of the non-transversal intersection curve.

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* Corresponding author.
E-mail addresses: osmar@matematica.uftm.edu.br (O. Aléssio), mduldul@yildiz.edu.tr (M. Düldül), buduldul@yildiz.edu.tr (B. Uyar Düldül), nhabdeal2002@yahoo.com (N.H. Abdel-All), sayed_badr@ymail.com (S.A.-N. Badr).

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1. Introduction

The surface–surface intersection problem which focuses on determining the intersection curve of two surfaces is an important topic in CAD, CAGD studies. For such problems, depending upon the parametric or implicit equations of the given surfaces, there exists three kinds of intersection types with two different cases in Euclidean 3-space $\mathbb{E}^3$. The intersection is called transversal intersection when the normal vectors of the intersecting surfaces are linearly independent at intersection point, and is called non-transversal when they are linearly dependent. The linear independence of the normal vectors enables to find the tangent vector of the intersection curve by a vector product. However, this situation is not valid for non-transversal intersections. For that reason, related with this problem, several methods in which transversal intersections attracted much attention have been given by different authors (see e.g. [1–11]). Recently, this problem is extended into higher-dimensional spaces (see [12] for Euclidean n-space).

In Euclidean 4-space $\mathbb{E}^4$, there exists four kinds of intersection types. Transversal intersections in $\mathbb{E}^4$ were studied by [11,13–18]. Non-transversal intersection of three hypersurfaces with the normal vectors $\mathbf{N}_i$ occurs in two different cases:

Case 1 (almost tangential intersection): $dim\{span\{\mathbf{N}_1(p), \mathbf{N}_2(p), \mathbf{N}_3(p)\}\} = 2$

Case 2 (tangential intersection): $dim\{span\{\mathbf{N}_1(p), \mathbf{N}_2(p), \mathbf{N}_3(p)\}\} = 1$.

Recently, we study the non-transversal intersection of parametric–parametric–parametric (PPP) hypersurfaces [19] and the non-transversal intersection of implicit–implicit–parametric (IIP), implicit–parametric–parametric (IPP) hypersurfaces [20] in $\mathbb{E}^4$ (we review the previous studies given for intersection problems in these publications).

Abdel-Aziz et al. [21] studied the differential geometry properties of tangential intersection curve of three implicit hypersurfaces (III) in $\mathbb{E}^4$. They consider the intersection curve γ as a curve parametrized by the variable x_1, i.e. $\gamma(x_1) = (x_1, x_2(x_1), x_3(x_1), x_4(x_1))$. This consideration restricts the applicability of their method to all non-transversal intersection problems (e.g. if their method is applied to the intersection problem given in our Example 4, we cannot obtain even the first curvature). They also analyzed only the tangential case (see Table 1).

In this paper, we have studied the two cases (tangential and almost tangential intersection). Our both approaches can be applied to any non-transversal intersection of three implicit hypersurfaces in Euclidean 4-space. Also, our methods contain some remarks showing what can be done when we encounter with vanishing linear equations. For that reason, our methods not only differ from the method given by [21] but also give a complete answer for such intersection problems.

In Section 2 we introduce some notations and reviews of the differential geometry of curves and hypersurfaces in $\mathbb{E}^4$. In Section 3, we derive formulas to compute the Frenet apparatus of the non-transversal intersection curve of three implicit hypersurfaces. In Section 4, by considering a parametric curve α whose Frenet apparatus are known from [22], we first find three implicit hypersurfaces on which α lies, and then apply our method to compare the results. To check all cases, some other examples are also given. Finally, in Section 5, we present our concluding remarks.

2. Review of differential geometry

2.1. Notation and cross product in 4-space

The notation for the derivative of a curve in relation to the arc-length s is $\alpha'(s) = \frac{d\alpha}{ds}, \alpha''(s) = \frac{d^2\alpha}{ds^2}, \alpha'''(s) = \frac{d^3\alpha}{ds^3}, \ldots, \alpha^{(n)}(s) = \frac{d^n\alpha}{ds^n}$.