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Random search of stable member in a matrix polytope

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A B S T R A C T

Given a matrix polytope we consider the existence problem of a stable member in it. We suggest an algorithm in which part of uncertainty parameters is chosen randomly. Applications to affine families and 3×3 interval families are considered. A necessary and sufficient condition for the existence of a stable member is given for general interval families with nonnegative off-diagonal intervals.

1. Introduction

Let $A = (a_{ij})$ be an $n \times n$ real or complex matrix. If all eigenvalues of the matrix A lie in the open left half-plane, the matrix A is called Hurwitz stable.

In many practical problems the entries a_{ij} of A are not known and in this case, the family

$$A(q) = \begin{pmatrix} a_{11}(q) & a_{12}(q) & \cdots & a_{1n}(q) \\ a_{21}(q) & a_{22}(q) & \cdots & a_{2n}(q) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}(q) & a_{n2}(q) & \cdots & a_{nn}(q) \end{pmatrix}, \quad (q \in Q)$$

is under consideration where the uncertainty vector q belongs to a box

$$Q = \{(q_1, q_2, \ldots, q_l)^T \in \mathbb{R}^l : q_i^- \leq q_i \leq q_i^+, i = 1, 2, \ldots, l\}.$$

Consider the family

$$\mathcal{A} = \{A(q) : q \in Q\}. \quad (1)$$

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The family (1) is said to be an interval matrix family if each component of q enters into only one entry $a_{ij}(q)$ of $A(q)$, i.e.

$$\mathcal{A} = \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \tag{2}$$

where $q_{11} \in [a_{11}, b_{11}], \ldots, q_{nn} \in [a_{nn}, b_{nn}]$.

The family (1) is called an affine family if $A(q) = A_0 + q_1A_1 + q_2A_2 + \cdots + q_lA_l$.

The family $\mathcal{A}$ is said to be robustly stable if every matrix in this family is Hurwitz stable. Robust stability of polynomial matrix families has been considered in [1–3].

A function $f : Q \to \mathbb{R}$ is said to be multilinear if it is affine-linear with respect to each component of $q \in Q$.

The following theorem expresses the well-known property of a scalar multilinear function defined on a box.

Theorem 1 ([4,5]). *Suppose that $Q \subset \mathbb{R}^l$ is a box with extreme points q^i, $(i = 1, 2, \ldots, k)$ and $f : Q \to \mathbb{R}$ is multilinear. Then both the maximum and the minimum of f are attained at extreme points of Q. That is,*

$$\max_{q\in Q} f(q) = \max_i f(q^i)$$

$$\min_{q\in Q} f(q) = \min_i f(q^i).$$

Theorem 1 can be used to find the range interval of a scalar multilinear function $f : Q \to \mathbb{R}$ defined on a box Q. The Cartesian product of these scalar intervals gives outer approximation of the range of a vector multilinear function.

A parameter dependent polynomial $p(s, q) = a_0(q)+a_1(q)s+a_2(q)s^2+\cdots+a_n(q)s^n$ is called multilinear if all coefficient functions $a_i(q)$ $(i = 0, 1, \ldots, n)$ are multilinear.

Consider the following question: Given a matrix family $\{A(q) : q \in Q\}$ is there a stable matrix in it? This problem is important in control theory [6]. For example, for the linear system $\dot{x} = Ax + Bu$ the feedback $u = Kx$ gives the closed loop system $\dot{x} = (A+BK)x$. Is there a feedback K such that the obtained system is asymptotically stable? (that is $A+BK$ is stable). Treating the entries of K as the parameter q, the problem can be reduced to the above one.

The existence of a stable member in a matrix polytope and other related problems has been considered in many works (see [6–10] and references therein).

In [6,8,9] universal numerical methods have been proposed to minimize the spectral abscissa function

$$\eta(q) = \max_k \mathrm{Re}\lambda_k(A(q)). \tag{3}$$

The function $\eta(q)$ is non-Lipschitz and non-convex and the corresponding minimization algorithms may get stuck in a local minimum (see Example 2).

In this work we propose random searching algorithm of a stable member for special classes of matrix families. Our approach is a modification of the Monte-Carlo method [11] since part of uncertainties is selected randomly and the remaining part deterministically. We give the applications of this algorithm to affine and 3×3 interval families.

In this paper

(i) For a one-parameter nonlinear $n \times n$ family whole stability region is determined (see Example 1). For a multiparameter family, random search is proposed. It can be considered a nonlinear version (see Example 3) of D-decomposition method from [6,9,10].
(ii) For 3×3 interval family with constant diagonals whole stability region is determined (see Examples 5 and 6). In the case of nonconstant diagonals, random search of a stable member is proposed.
(iii) For $n \times n$ interval family with nonnegative off-diagonal intervals a simple necessary and sufficient condition for the existence of a stable member is obtained (Section 4).

It should be noted that differently from [6,8,9] the results of Sections 2.2 and 3.1 are applicable for lower dimensional systems. For the comparison with the known methods see Section 5 and the end of Examples 1 and 4.

2. Stable member in nonlinear matrix families

In this section we consider the problem of finding a stable member in matrix families. Firstly an one parameter nonlinear family is considered and by using the bialternate product of matrices all of the stable regions are determined. For a multiparameter family we suggest all parameters are chosen randomly with the exception of one chosen parameter, which reduces the problem to a one-parameter case. In this case our result can be considered as a nonlinear version of D-decomposition method from [6,9,10]. See Example 3.