



Computational analysis of elastic field of deep curved beams/rings using displacement-function equilibrium method

S. Reaz Ahmed*, Abhishek Kumar Ghosh

Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh

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ABSTRACT

A new computational approach is proposed for determining the elastic field of deep curved beams/rings, which is based on a displacement-function equilibrium method. The displacement function is defined in terms of the radial and circumferential components of displacements, which is governed by a single partial differential equation of equilibrium. Solution of the elastic field is obtained in terms of the displacement function, in which both the uniform and the mixed mode of boundary conditions are treated with equal sophistication. The application of the single function computational method is demonstrated through the numerical solutions of a number of problems of curved beams and rings with mixed boundary conditions. The soundness and accuracy of the investigation are verified through the comparison of results with those obtained by conventional computational and analytical approaches available.

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1. Introduction

Although the analysis of stresses in structural members with initial curvature has become a classical subject in the field of structural mechanics, the literature shows that they are being constantly looked into with improved sophistication in their method of solution. Originally two theoretical approaches were available for stress analysis of curved beams and bars, which are Winckler's strength-of-materials theory [1] and Timoshenko's elasticity formulation [2]. Winckler's theory mainly gives the analytical expression for the circumferential stress in a curved beam, which is, however, not adequate when the beam is thick. The more refined Timoshenko's theory has served as the standard of comparison for most of the attempted modifications of Winckler's theory. A general solution involving infinite series to the distribution of stresses in circular ring compressed by two forces acting along a diameter was given by Chianese and Erdlac [3]. Bagci [4] presented a unified strength of materials solution for stresses in curved beams and rings which considers the curvature effect on both moment and force loadings. The elasticity solution for curved beams and rings with exponential and 'T' cross section was presented by Bagci [5], which is however identified to be an approximation because the state of stress was arbitrarily assumed to be plane (see Ref. [6]). A plane stress analysis based on stress function formulation [2] was presented by Tutuncu [7] for the stress and displacement distributions in polar orthotropic curved beams possessing narrow constant cross sections. Recently, Sloboda and Honarmandi [8] have developed an elasticity based method for stress analysis of curved beam of non-rectangular cross sections, which however accepts boundary conditions only in terms of boundary loadings (stresses).

* Corresponding author.

E-mail address: srahmed@me.buet.ac.bd (S. Reaz Ahmed).

As far as the elasticity based methods are concerned, the stress function formulation [2] has been successfully used for the analysis of curved beams/rings, in which, however, the physical conditions on the bounding surfaces are prescribed only in terms of loading parameters. Boundary restraints specified in terms of radial or circumferential components of displacement/strain cannot be satisfactorily imposed on the stress function. The direct displacement-parameter approach, on the other hand, involves finding two displacement parameters (u_r and u_θ) from two differential equations of equilibrium. However, simultaneous determination of two unknowns satisfying two second-order partial differential equations with variable coefficients becomes extremely difficult especially when the boundary conditions are expressed as a mixture of boundary restraints and loadings (third kind). Since most of the practical problems of structural mechanics are of mixed boundary-value type, the conventional approaches are many times found to be inadequate for their accurate and reliable analysis [9–11].

Most of the structural components in practical applications are of mixed-boundary-value type, as they are always used with various supports, stiffeners, guides, and restraints. The analysis and design of these structures are mainly accomplished by numerical methods. Among the various approaches, the finite-element method (FEM) and the finite-difference method (FDM) are the major numerical methods in practical use. The FEM has received widespread applications in various aspects of structural analysis, especially for curved structures, some of which are cited in Refs. [12,13]. It has been pointed out that there are uncertainties in accurate and reliable prediction of surface stresses by the conventional computational approaches [14,15]. The errors arising in the curved finite elements which under goes both flexure and membrane deformations were pointed out by Gangan [16]. It was there reported that an error of special nature would involve if the membrane strain fields are not consistently interpolated with terms from the two independent field functions, and the associated physical phenomenon was termed as membrane locking.

The accuracy of FDM, especially, in predicting critical stresses in structural components, like beams has been verified to be much higher than the corresponding accuracy of FE analysis [17,18]. Some of the recent researches and developments have generated renewed interests in using FDM for stress analysis of both two- and three-dimensional structural components [19–22]. The performance of a potential-function based FDM in predicting stresses at the regions of transition of boundary conditions has been verified to be more justified compared to that of conventional computational method [23]. It can be mentioned that the application of FDM for boundary value problems in various fields of physical sciences and engineering, especially those governed by second-order partial differential equations is constantly coming up in the literature with improved sophistication in the method of solution [24–28]; however, serious attempts to accurate and reliable solution of higher order partial differential equations, especially with variable coefficients and mixed mode of boundary conditions have hardly been made in the past.

An interpolation based computational technique has been reported to model the irregular boundary shapes of elastic bodies using a uniform rectangular mesh network [29]. However, the use of curvilinear coordinate system for analyzing the elastic field of structural problems of curved geometries, like curved bars/beams, arches, rings, etc. shows more flexibility as well as advantages especially in modeling the boundary shapes and the associated boundary conditions. Because, the management of boundary shapes and conditions can be realized without adopting interpolation or extrapolation like approximations, thereby providing more accurate prediction of stresses, especially at the surfaces, which are in general the sections of critical stresses for structural components. The present paper describes a new computational analysis of the elastic field of curved structural members with mixed boundary conditions, namely, curved beams, deep arches, rings, etc. using a displacement function based computational method. The function is defined in terms of the radial and circumferential components of displacements, and is governed by a single fourth-order elliptic partial differential equation of equilibrium. A finite-difference computational scheme is proposed to obtain the displacement function solution for the stress problems with initial curvature, which eventually offers a tremendous saving in computational effort from those of conventional approaches, as a single variable is evaluated at each nodal point of the computational domain [20]. Moreover, the present scheme allows us to handle both the uniform as well as mixed mode of boundary conditions with equal sophistication. Finally, the displacement-function solutions of a number of problems of curved beams and rings are discussed in light of comparison made with those obtained by the standard computational method as well as analytical solutions where available.

2. Displacement function equilibrium method (DFEM)

2.1. General relations

The development of a displacement function formulation for the boundary-value stress problems of solid mechanics with initial curvature (curved geometries) is described in this section. Fortunately most of the practical problems of stress analysis can reasonably be resolved into two-dimensional ones following one of the two simplifying assumptions, namely, the plane stress and plane strain conditions. With reference to a polar coordinate system (r, θ), in absence of body forces, the three governing differential equations for isotropic materials, in terms of stress variables σ_r, σ_θ and $\tau_{r\theta}$ under both plane stress and plane strain conditions are as follows [2]:

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1a)$$

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