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Meshless method and convergence analysis for 2-dimensional Fredholm integral equation with complex factors

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ABSTRACT

In this paper, the meshless method is presented for numerically solving the 2-dimensional Fredholm integral equation with complex factors. First, the existence and uniqueness of solution are proved by the fixed point theorem. Second, the approximation solution method of the radial basis function interpolation is given. The results of convergence analysis and error estimation are also obtained. Some numerical examples are presented to verify the validity and applicability of the method.

1. Introduction

Some problems in many fields of science can be summed up in solving Fredholm integral equation such as mathematics, physics, biology and etc. So solving the integral equation has received the widespread attention. Many numerical methods are used for numerical solution of integral equation, for instance, collocation method [1], iterative method [2–4], least squares approximation method [5], spectral method [6] and so on. And the study of 2-dimensional Fredholm integral equation numerical solution is also widespread, such as the Kansa's collocation method which based on the radial basis function [7,8]. However, few papers reported application meshless method to solve the 2-dimensional Fredholm integral equation with complex factors.

In this study, we are concerned on a class of 2-dimensional integral equation with complex factors of the form

$$u(x,y) - A(x,y)u(\alpha(x),\beta(y)) = f(x,y) + \lambda \int_{\Omega} k(x,y,\xi,\eta)u(\alpha(\xi),\beta(\eta))d\xi d\eta, \tag{1}$$

where $\Omega \in \mathbb{R}^2$ and Ω is bounded closed domain, $A(x,y), f(x,y), k(x,y,\xi,\eta), \alpha(x), \beta(y)$ are the appropriate smooth known functions, $u(x,y)$ is the unknown function on Ω. In particular, when $\alpha(x)$, $\beta(y)$ are first-order polynomials, the Eq. (1) is reduced to 2-dimensional functional integral equation with double proportional delays.

The radial basis function method is one of the meshless method which was produced in the 1950s and commonly used to study the high dimensional problem. The relevant theories of the radial basis function have been investigated extensively,

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see [9–11]. In this work, the radial basis function method is used to obtain the approximate solution of a class of the 2-dimensional Fredholm integral equation with complex factors. The existence and uniqueness of solution for the Eq. (1) under certain conditions will be presented in the next section.

2. The existence and uniqueness of solution

In this section, we discuss the existence and uniqueness of solution for Eq. (1) under some appropriate conditions. Before this, we recall the following preliminaries.

Let (X, d) be a metric space, $T : X \to X$ is a mapping, if there exists a constant $\alpha : 0 \le \alpha < 1$, such that $d(Tx, Ty) \le \alpha \cdot d(x, y)$, for any $x, y \in X$. Then the mapping T is called a contraction mapping on X. If (X, d) be a complete metric space, $T : X \to X$ be a contraction mapping. Then there exists a unique fixed point of mapping T on X.

Throughout this paper, we consider the complete metric space $(C(\Omega), d)$, where $d(u_1, u_2) = \|u_1 - u_2\|_\infty$, for all $u_1, u_2 \in C(\Omega)$, and we assume that

$$Tu = A(x, y)u(\alpha(x), \beta(y)) + f(x, y) + \lambda \int_\Omega k(x, y, \xi, \eta)u(\alpha(\xi), \beta(\eta))d\xi d\eta. \tag{2}$$

Moreover, let Eq. (1) satisfies the following conditions:

(i) $(\alpha(x), \beta(y)) \in \Omega$, as $(x, y) \in \Omega$.
(ii) $\|A(x, y)\|_\infty + |\lambda| \max_{(x,y)\in\Omega} \int_\Omega |k(x, y, \xi, \eta)|d\xi d\eta = \gamma < 1$.

Then we have the following theorem.

Theorem 1. *Consider the above conditions* (i)–(ii)*, then Eq.* (1) *exists a unique solution.*

Proof. First, we prove T is a contraction mapping. One can easily see that

$$\begin{aligned} d(Tu_1, Tu_2) &= \Big\| A(x, y)(u_1(\alpha(x), \beta(y)) - u_2(\alpha(x), \beta(y))) \\ &\quad + \lambda \int_\Omega k(x, y, \xi, \eta)\big[u_1(\alpha(\xi), \beta(\eta)) - u_2(\alpha(\xi), \beta(\eta))\big]d\xi d\eta \Big\|_\infty \\ &\le (\|A(x, y)\|_\infty + |\lambda| \int_\Omega |k(x, y, \xi, \eta)|d\xi d\eta) \cdot \|u_1 - u_2\|_\infty \\ &\le \gamma \cdot \|u_1 - u_2\|_\infty. \end{aligned}$$

Since $\gamma < 1$, then T is a contraction mapping. There exists a unique fixed point u such that $Tu = u$. So the proof is completed.

3. The radial basis function interpolation

In this section, some basic concepts and results of the radial basis function interpolation are introduced. The problem is that the nth observational data $\{(x_i, y_i, u(x_i, y_i))\}_{i=1}^n$ of the function $u(x, y)$ on Ω are known, one aims to obtain $u_n(x, y)$ such that

$$u_n(x_i, y_i) = u(x_i, y_i), \quad i = 1, 2, \ldots, n.$$

The base of the radial basis function are $\varphi_i(x, y) = \varphi([(x - x_i)^2 + (y - y_i)^2]^{\frac{1}{2}})$, $i = 1, 2, \ldots, n$, some common basis functions are the following:

(i) Gauss distribution function of the method of kriging: $\varphi(r) = e^{-a^2 r^2}$.
(ii) Multi-Quadric function of Hardy: $\varphi(r) = (c^2 + r^2)^b$ and Inverse Multi-Quadric function: $\varphi(r) = (c^2 + r^2)^{-b} (b > 0)$.
(iii) Thin plate splines of Duchon: $\varphi(r) = r^{2K-d} \log r$.

The radial basis function interpolation of $u(x, y)$ and $u(\alpha(x), \beta(y))$ have the form

$$\begin{cases} u_n(x, y) = \sum_{i=1}^n \lambda_i \varphi_i(x, y), \\ u_n(\alpha(x), \beta(y)) = \sum_{i=1}^n \lambda_i \varphi_i(\alpha(x), \beta(y)). \end{cases} \tag{3}$$

In the following, we will give the convergence result of the radial basis function interpolation which can be found in [11].

$$\|u(\alpha(x), \beta(y)) - u_n(\alpha(x), \beta(y))\|_\infty \le \|u(x, y) - u_n(x, y)\|_\infty \le Ch^{\frac{e}{2}}, \tag{4}$$