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# Damping optimization over the arbitrary time of the excited mechanical system

Ivana Kuzmanović, Zoran Tomljanović *, Ninoslav Truhar

*Department of Mathematics, J. J. Strossmayer University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia*

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ABSTRACT

In this paper, we consider damping optimization in mechanical system excited by an external force. We use optimization criteria based on minimizing average energy amplitude and average displacement amplitude over the arbitrary time. As the main result we derive explicit formulas for objective functions. These formulas can be implemented efficiently and accelerate optimization process significantly, which is illustrated in a numerical example.


## 1. Introduction

The main problem which we consider is an optimization of damping in the parameterized mechanical system described with the system of ordinary differential equations

$$M\ddot{x}(t) + D(v)\dot{x}(t) + Kx(t) = g(t), \tag{1}$$

where mass and stiffness matrices $M$ and $K$ are positive definite real matrices of order $n$ and the vector $g(t) \in \mathbb{R}^n$ corresponds to an external force. The damping matrix $D(v) \in \mathbb{R}^{n\times n}$ is a positive semidefinite matrix which depends on damping positions and parameter $v \in (0, \infty)$, called viscosity.

The problem of damping optimization belongs to the very interesting and active research area which has several different approaches. The usage of an appropriate approach strongly depends on whether the system (1) is stationary ($g(t) = 0$) or non-stationary ($g(t) \neq 0$). Damping optimization in both cases is very demanding since it requires a large number of function evaluations. Especially, problem of optimization of damping positions with viscosities still does not have satisfying solution.

The case of the stationary system has been widely investigated during the last two decades. We will emphasize several references: [1–9]. The more detailed description of these references can be found in [10].

On the other hand, the problem of non-stationary systems have been considered in [10–12].

There are several different (damping) optimization criteria and the most common ones are based on the asymptotic approach or approach in which the damping criterion is based on the infinite time scale. For example in [12–16], the optimal

* Corresponding author.
*E-mail address:* ztomljan@mathos.hr (Z. Tomljanović).

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displacement or optimal dampers positions are based on the criterion which considers asymptotical behavior. On the other hand in [10], the optimization criteria are defined over the basic period of the periodic external force $g(t)$.

The main results of this paper are natural extensions of the considerations started with the paper [10], with a more general setting. Similarly as in the paper [10], here we consider the problem of damping optimization for the system (1) with the respect to optimization criteria:

- *average energy amplitude*
- *average displacement amplitude*.

The main generalization in this paper is that we use the optimization criteria defined over the arbitrary time which is, with respect to considered time, the most general case. This yield that results from [10] can be obtained by setting considered time to a basic period. Consideration of an arbitrary time is much more demanding since it requires much more additional terms (which vanish in the case of period).

The reason for consideration of the arbitrary time lies in the fact that in many situations it is of our interest to observe system behavior in time much smaller or larger than the period of external force. Moreover, consideration of arbitrary time allows us to optimize damping with respect to time. Considered optimization can be done efficiently using results of this paper since we will derive explicit formulas for calculation of above mentioned criteria. Explicit formulas provide an accurate damping optimization with significant time acceleration. Acceleration of optimization procedure is a very important issue since damping optimization is known as a very demanding task.

To summarize, the main results of this paper are new explicit (with respect to optimization parameter) formulas for objective functions in criteria average energy amplitude over the arbitrary time and average displacement amplitude over the arbitrary time. Obtained formulas allow efficient damping optimization as we will illustrate through the paper.

## 2. Damping optimization

### 2.1. Preliminaries

Since the external force $g(t)$ can be approximated with a Fourier series, we assume that it is given as the sum of sine and cosine functions. Thus, we consider a model (1) of the form

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = \sum_{j=1}^{p} f_j^a \cos(\widehat{\omega}_j t) + f_j^b \sin(\widehat{\omega}_j t). \tag{2}$$

Damping matrix $D$ is a matrix with small rank $r \ll n$ and it can be written in a form $D = D_r D_r^*$ where $D_r \in \mathbb{R}^{n\times r}$ describes the geometry of damping positions.

Using standard results from the theory of ordinary differential equations, it is well known that the solution of (2) is of the form

$$x(t) = \sum_{j=1}^{p} x_j(t) \tag{3}$$

where summands $x_j$ are solutions of differential equations

$$M\ddot{x}_j(t) + D(v)\dot{x}_j(t) + Kx_j(t) = f_j^a \cos(\widehat{\omega}_j t) + f_j^b \sin(\widehat{\omega}_j t) \tag{4}$$

for $j = 1, \ldots, p$.

As it has been shown in [11], after substitution $x_j(t) = x_j^a \cos(\widehat{\omega}_j t) + x_j^b \sin(\widehat{\omega}_j t)$ and by introducing complex quantities

$$x_j^0 = x_j^a - i x_j^b, \qquad f_j^0 = f_j^a - i f_j^b \quad \text{for } j = 1, \ldots, p \tag{5}$$

the system (4) is equivalent to

$$(i\widehat{\omega}_j I - A) y_j^0 = F_j^0, \quad \text{where } y_j^0 = \begin{bmatrix} L_1^* x_j^0 \\ i\widehat{\omega}_j L_2^* x_j^0 \end{bmatrix}, \qquad F_j^0 = \begin{bmatrix} 0 \\ L_2^{-1} f_j^0 \end{bmatrix}, \tag{6}$$
$$A = \begin{bmatrix} 0 & L_1^* L_2^{-*} \\ -L_2^{-1} L_1 & -L_2^{-1} D L_2^{-*} \end{bmatrix}, \qquad K = L_1 L_1^*, \qquad M = L_2 L_2^*$$

and $y_j(t) = y_j^0 e^{i\widehat{\omega}_j t}$ is a solution of

$$\dot{y}_j(t) = A y_j(t) + F_j^0 e^{i\widehat{\omega}_j t} \quad \text{for } j = 1, \ldots, p. \tag{7}$$

From Eq. (7) it follows that

$$y(t) = \sum_{j=1}^{p} y_j(t) \tag{8}$$