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Data envelopment analysis in the absence of convexity: Specifying efficiency status and estimating returns to scale

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HIGHLIGHTS

- A DEA method for identifying FDH-efficient DMUs.
- Two DEA models for the estimation of right and left RTS of FDH-efficient DMUs.
- A DEA method for determining MPSS in FDH models.
- Presentation of two examples to illustrate the use of our proposed approach.
- Making comparisons among our RTS method and other RTS methods in FDH models.

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ABSTRACT

The first purpose of this study is to present new DEA (data envelopment analysis) approaches that distinguish the efficiency status (efficient or inefficient) of decision-making units (DMUs) and estimate right- and left-hand returns to scale (RTS) of efficient DMUs in free disposal hull (FDH) models. The second one is to estimate most productive scale size (MPSS) in FDH models. The proposed RTS method has three preferences with respect to the previous RTS methods in this context. First one is that it is capable of identifying all FDH-efficient DMUs, whereas the previous methods may not be able to specify all FDH-efficient DMUs. Besides, the second one is to estimate right- and left-hand RTS of FDH-efficient DMUs, while the former methods are only capable of estimating RTS of FDH-efficient DMUs. Moreover, the third one is that it is always able to estimate right- and left-hand RTS, correctly, whereas the previous methods are sometimes capable of estimating RTS, correctly. These issues are the motivations for creating this current research. Finally, a numerical example and an empirical application are exhibited for illustrating and comparing the proposed method with the former methods. Then, some consequences are provided based upon the obtained results and also some suggestions for future works are presented. Proofs of some theorems and lemmas are given in the Appendix.

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1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for evaluating the performance/efficiency of decision-making units (DMUs) which was first proposed by Charnes, Cooper, and Rhodes (CCR) [1]. Then, Banker, Charnes, and Cooper (BCC) [2] presented a variable returns to scale (VRS) version of the CCR model. One of the important topics in DEA literature is free disposal hull (FDH) models which were first introduced by Deprins et al. [3]. FDH models rely on the sole assumption that production possibility set (PPS) satisfies free disposability and also these models ensure that efficiency evaluations are relative to actually observed performances only.

Hitherto, in the context of evaluating efficiency and also estimating returns to scale of DMUs, several DEA approaches have been proposed by many researchers [4–18]. One of the vital topics in FDH models is to estimate right- and left-hand returns to scale and determine most productive scale size (MPSS), that so far, there is no article in this connection in DEA literature. It is noteworthy that, there are only few articles which introduce some DEA approaches for estimating returns to scale in FDH models [19–23].

Kerstens and Vanden Eeckaut (K & VE) [19] introduced a method based on goodness-of-fit for estimating returns to scale in FDH models using non-parametric deterministic technologies. In order to evaluate the efficiency of the target DMU in CRS (constant returns to scale), NIRS (non-increasing returns to scale), and NDRS (non-decreasing returns to scale), K & VE suggested solving three mixed integer nonlinear programming problems.

Moreover, K & VE's approach was improved by Podinovski [20] via solving three equivalent mixed integer linear programming problems. Solving K & VE's, and Podinovski's approaches can be onerous from a computational point of view.

In addition, Soleimani-damaneh et al. (S-det al.) [21], Soleimani-damaneh and Reshadi (S-d & R) [22], and Soleimani-damaneh and Mostafaee (S-d & M) [23] introduced some methods based on the evaluation of certain ratios without solving any programming models to estimate returns to scale of DMUs.

In this current study, a DEA method is first presented to distinguish efficiency status (efficient or inefficient) of DMUs in FDH models. Then, two new DEA models are respectively proposed in order to estimate right- and left-hand returns to scale of the FDH-efficient DMUs. Furthermore, some theorems are presented to identify MPSS in FDH models.

It is necessary to mention that, the proposed RTS approach has the following advantages with respect to K & VE's, Podinovski's, S-d et al.'s, S-d & R's, and S-d & M's RTS approaches in DEA literature.

- It is capable of identifying all FDH-efficient DMUs whereas K & VE's, Podinovski's, and S-d et al.'s, S-d & R's, and S-d & M's approaches may not be able to identify all FDH-efficient DMUs and their approaches identify some of the FDH-efficient DMUs.
- It is capable of estimating right- and left-hand returns to scale of FDH-efficient DMUs whereas K & VE's, Podinovski's, S-d et al.'s, S-d & R's, and S-d & M's approaches are only capable of estimating returns to scale of FDH-efficient DMUs.
- It is always capable of estimating right- and left-hand returns to scale, correctly, whereas K & VE's, Podinovski's, S-d et al.'s, S-d & R's, and S-d & M's approaches are sometimes capable of estimating returns to scale, correctly.

It is noticeable that FDH-inefficient DMUs have more than one projection on the non-convex PPS frontier of FDH models, for this reason by using the proposed method, various right- and left-hand returns to scale can be acquired for their projections.

Therefore, in the real world applications of DEA, our proposed RTS approach is more accurate than K & VE's, Podinovski's, S-d et al.'s, S-d & R's, and S-d & M's RTS approaches.

The rest of this article is organized as follows. Section 2 is the short background on the previous RTS approaches such as K & VE's, Podinovski's, S-d et al.'s, and S-d & R's approaches. The proposed RTS method is presented in Section 3. Section 4 introduces some theorems to determine MPSS. In Section 5, two illustrative examples are presented. Section 6 includes concluding remarks along with future research agendas.

2. Preliminaries and literature review

Assume a set of n DMUs, i.e. $\{DMU_j \,|j = 1, 2, \ldots, n\}$, where DMU_j uses m different inputs $X_j = (x_{1j}, \ldots, x_{ij}, \ldots, x_{mj})^t \geq_{\neq} 0$ to produce s different outputs $Y_j = (y_{1j}, \ldots, y_{rj}, \ldots, y_{sj})^t \geq_{\neq} 0$. In this research, the superscript "t" represents a vector transpose. Likewise, production possibility set (PPS) is specified as $PPS = \left\{ \binom{X}{Y} |Y \text{ can be produced by } X \right\}$. The following PPS is obtained under the assumption of variable RTS (VRS):

$$T_V = \left\{ \begin{pmatrix} X \\ Y \end{pmatrix} \middle| \sum_{j=1}^{n} \lambda_j X_j \leq X,\ \sum_{j=1}^{n} \lambda_j Y_j \geq Y,\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\ (j = 1, 2, \ldots, n) \right\}. \tag{1}$$

Hereafter, in this study, symbol "$*$" indicates optimal solution values.