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Gibbs sampling approach to regime switching analysis of financial time series

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ABSTRACT

We will introduce a Monte Carlo type inference in the framework of Markov Switching models to analyse financial time series, namely the *Gibbs Sampling*. In particular we generalize the results obtained in Albert and Chib (1993), Di Persio and Vettori (2014) and Kim and Nelson (1999) to take into account the switching mean as well as the switching variance case. In particular the volatility of the relevant time series will be treated as a state variable in order to describe the abrupt changes in the behaviour of financial time series which can be implied, e.g., by social, political or economic factors. The accuracy of the proposed analysis will be tested considering financial dataset related to the U.S. stock market in the period 2007–2014.

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1. Introduction

Dealing with financial time series is in most cases a non-deterministic task, since a common assumption is the presence of a stochastic error in the empirical datum, and the phenomenon could depend both from observed and unobserved variables, the latter usually called *state variables*. The change of relevant states during time gives rise to the so called *regime switching* dynamic which is governed by specific laws assumed (in literature) to be either deterministic or stochastic.

In this work we are going to deal only with *discrete state-space models* (DSSM) with *Markovian switching* for a specific financial framework taken into account. The aim will be to make a complete quantitative analysis from the rough time series $\{y_t\}_{t=1}^{T}$, being T a positive integer representing the expiration date or the number of available observations. The resulting system will be of the following form:

$$\begin{cases} y_t = f(S_t, \theta, \psi_{t-1}) \\ S_t = g\left(\tilde{S}_t, \psi_{t-1}\right) \\ S_t \in \Lambda \end{cases} \tag{1}$$

where $\psi_t := \{y_k : k = 1, \dots, t\}$, θ is the vector of the model's parameters, Λ represents the set of the all the possible states, g is the *state-switching* law, namely a function of the past states and the observations until the previous time, while f is the function $f : \Lambda \times \mathbb{R}^k \times \mathbb{R}^{t-1} \to \mathbb{R}$, where k is the number of the *descriptive parameters*, which returns the actual value of the time series at time t. We would like to underline that this class of models is widely used, e.g., in engineering, physics, etc., where they have been implemented to, e.g., study stochastic resonance phenomena, see [1], to underline the relation between corn and oil prices through DSSM, see [2], or to optimal control problem for DC–DC converter systems,

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see [3]. Concerning our analysis, we are going to embed system (1) in the case of a serially uncorrelated time series with Markovian regime switching and a four-dimensional state space. Markov hypothesis is a standard choice in financial time series analysis, while for the choice of the dimensionality we refer to [4], where the authors underline the necessity of distinguishing both the *high risk state* and the *structural break state*. Therefore we obtain the following model:

$$\begin{cases} y_t &= \mu_{S_t} + \epsilon_t, \quad t = 1, 2, \dots, T \\ \epsilon_t &= \text{i.i.d. } \mathcal{N}(0, \sigma_{S_t}^2), \\ \mu_{S_t} &= \mu_1 S_{1,t} + \mu_2 S_{2,t} + \mu_3 S_{3,t} + \mu_4 S_{4,t}, \\ \sigma_{S_t} &= \sigma_1 S_{1,t} + \sigma_2 S_{2,t} + \sigma_3 S_{3,t} + \sigma_4 S_{4,t}, \\ S_t &\in \{1, 2, 3, 4\} \\ p_{ij} &:= \mathbb{P}(S_t = j | S_{t-1} = i) \quad i, j = 1, 2, 3, 4, \end{cases} \tag{2}$$

where $S_{j,t}$ is the characteristic function for the event *being in state j at time t*. We will refer to (2) as to a serially uncorrelated Markov Switching Model (MSM).

Our quantitative study is now shifted to an identification problem with the task of finding the switching probabilities p_{ij} and the *mean–variance* couples $[\mu_m, \sigma_m]$ that describe each state in the DSSM shown in (2). A classical approach to this identification problem is the well known *Maximum Likelihood approach*, which has been exhaustively investigated in [5], to which we refer also for details about the choice of using the *Hamilton filter*.

2. Bayesian inference

Our main problem is to infer on parameters that are subject to stochastic behaviour. In this framework a smart solution is to exploit the *Bayesian Inference* approach, that is a class of methods based on the Bayes' rule. The core of all these methods is the relation between likelihood functions and random variables, namely in order to explain the posterior distribution of a parameter, we write:

$$f\left(\theta|\hat{y}\right) = \frac{f\left(\hat{y}|\theta\right) f\left(\theta\right)}{f\left(\hat{y}\right)}, \tag{3}$$

where on the left hand side we have the joint posterior distribution of the parameters, while on the right hand side we have the product between the likelihood of the data and the prior distribution of the parameters divided by the marginal likelihood of the data (which can be considered constant). The latter suggests to focus the attention on the proportion

$$f\left(\theta|\hat{y}\right) \propto f\left(\hat{y}|\theta\right) f\left(\theta\right). \tag{4}$$

A good choice for the prior distribution would let us compute posterior distributions in an easy way, rather analytically. The latter is not a simple task, but we can exploit the results in [6] to recover that every member of the exponential family has conjugate priors. In particular, if prior and posterior distributions belong to the same family, we say that they are *conjugate distributions*, and the prior is called *conjugate prior for the likelihood*.

2.1. Conjugate distribution

Exploiting the fact that the model is subjected to constraints for the parameters, e.g. we want to have a value for the variance that is not negative, or we would like to preserve the possibility for the mean to take both positive and negative values, we first try to obtain the posterior distribution taken such constraints into account and then we go back to the prior we need to consider. Let us note that every member of the exponential family is endowed with a conjugate prior, hence it is reasonable to exploit such a set, in particular if n is the number of observations with $\tilde{y} = \{y_i\}_{i=1}^n$ being the dataset considered, we have the conjugation properties stated in the following subsections.

2.1.1. Bernoulli with unknown probability p

One basic feature we would like to deal with is the constraint on the inferred parameter that forces it to be a probability value, i.e. $0 \le p \le 1$. A probability distribution that ensures this feature is the *Beta distribution*, that is *self-conjugated*, which means that the prior and the posterior distributions are of the same kind. We can start by considering a random variable $X \sim \text{Bin}(1, p)$, hence X takes values on the set $\{0, 1\}$, $\mathbb{P}(X = 0) = p$ and $\mathbb{P}(X = 1) = 1 - p$. Inferring on p by the Bayesian method means considering p as a random variable and choosing a prior distribution that fits our constraints. One possible easy choice, which is the one we will later adopt, is to suppose that

$$f(p) \propto \text{Beta}(\alpha, \beta), \tag{5}$$

that leads to the posterior distribution of p given $\tilde{y}$:

$$f(p|\tilde{y}) = \text{Beta}\left(\alpha + \sum_{i=1}^n y_i, \beta + n - \sum_{i=1}^n y_i\right). \tag{6}$$

It is worth to mention that the relevance of the previously sketched framework will be clear in Section 3.1.1.

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