Journal of Computational and Applied Mathematics 300 (2016) 68–82

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Utility indifference pricing of derivatives written on industrial loss indices

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ARTICLE INFO

Article history:
Received 15 October 2014
Received in revised form 25 September 2015

MSC:
91B16
91G20
93E20
60J75

Keywords:
(Re-)Insurance
Catastrophe derivatives
Jump process
Random thinning
Utility indifference price

ABSTRACT

We consider the problem of pricing derivatives written on some industrial loss index via utility indifference pricing. The industrial loss index is modeled by a compound Poisson process and the insurer can adjust her portfolio by choosing the risk loading, which in turn determines the demand. We compute the price of a CAT (spread) option written on that index using utility indifference pricing and present numerical examples.

1. Introduction

It was recognized shortly after the Hurricane Andrew in 1992, then the most costly natural catastrophe in history, that events of this magnitude significantly stress the capacity of the insurance industry. On the other hand, the accumulated losses of those events are rather small relative to the US stock and bond markets. Thus, securitization offers a potentially more efficient mechanism for financing CAT losses than conventional insurance and reinsurance, see Cummins et al. [1].

The first contracts of this kind were launched by the Chicago Board of Trade (CBOT), which introduced catastrophe futures in 1992 and later introduced catastrophe put and call options. The options were based on aggregate catastrophe-loss indices compiled by Property Claims Services, an insurance industry statistical agent, see [2].

In the absence of a traded underlying asset, insurance-linked securities have been structured to pay-off on three types of variables: Insurance-industry catastrophe loss indices, insurer-specific catastrophe losses, and parametric indices based on the physical characteristics of catastrophic events. The first variant involves higher basis risk and less exposure to moral hazard than the second, the third variant tries to balance the two risks in a suitable way, cf. Cummins [2]. In this paper we solely concentrate on index-based derivatives.

A simple example of such a derivative is provided by the aforementioned call options on an insurance-industry catastrophe loss index. The variant introduced by CBOT was actually a call option spread, that is, a combination of a call option long and another call option short with a higher strike.

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http://dx.doi.org/10.1016/j.cam.2015.11.028
0377-0427/

A more popular type of catastrophe derivative is the CAT bond. This is a classical bond in which there is an option embedded which is triggered by a defined catastrophic event. In this paper we will again only consider those bonds where this catastrophic event depends on some industry-loss index, though in practice both of the other variants are of importance as well. From our point of view there is little difference between CAT bond and CAT option, since on evaluating a CAT bond we concentrate on the embedded option. There is however some danger of confusion regarding the role of buyer/seller with CAT bonds: The issuer of the bond actually buys the embedded option while the buyer of the bond sells the option.

For the issuer of a CAT bond – typically an insurance or reinsurance company – it serves as a reinsurance. On the other hand, the investor who buys the bond (and therefore sells an option) receives a coupon over the market interest and can, at the same time, diversify her risk by investing in a security whose payoff is largely uncorrelated with classical financial instruments.

Geman and Yor [3] analyze catastrophe options with payoff $(C(T)-K)^+$ where C is the aggregate claims process which is modeled by a jump–diffusion process. Cox [4] used a pure Poisson process to model the aggregate loss of an insurance company, and derived the pricing formula of CATEputs under the assumptions of constant arrival rates of catastrophic events. Jaimungal and Wang [5] used a compound Poisson process to describe the dynamic losses more accurately, but maintain the assumption of the constant arrival rate of claims.

We model the arrival of claims, which are accounted for by some industrial loss index, as a Poisson process with fixed arrival intensity. The underlying of the CAT derivative, the index, is itself not tradable. It therefore makes sense to use the method of indifference pricing via expected utility of Hodges and Neuberger [6] to price the derivative. A similar approach can be found in Egami and Young [7], where the authors used utility indifference pricing techniques to price structured catastrophe bonds. However, there is a big difference in our modeling of the hedging opportunity. In our setup this is done via adjusting the insured portfolio.

For catastrophic events, the assumption that the resulting claims occur at jump times of a Poisson process as adopted by most previous studies is not beyond justifiable critique. Therefore alternative point processes have been used to generate the claim arrival process. Lin et al. [8] proposed a doubly stochastic Poisson process, (also called "Cox process", see [4,9–12]) to model the arrival process for catastrophic events and derived pricing formulas of contingent capital. See also Fuita et al. [13] for arbitrage pricing of CAT bonds in such a context. Jaimungal and Chong [14] consider valuation of catastrophe derivatives when the rate of the claims is modulated by a Markov chain.

Charpentier [15] considers hedging of catastrophe derivatives with stocks whose jumps depend on catastrophic events and how to compute a utility indifference price in this setup.

Our study contributes to the literature by presenting a new approach to hedging a CAT derivative via adjustment of the insured portfolio, which in turn is done via adjusting the risk loading and an exogenously given demand curve. The main idea is that the loss in the portfolio of a single insurance company is necessarily correlated with an industrial loss index that includes the losses of that insurance. The introduction of the derivative has therefore an influence on the pricing policy of the insurance company.

It has been noted by Cummins [2] that the relatively low volume in the CAT derivatives market may in part be due to insufficient understanding of how these products may be hedged. Our paper gives a new perspective to the hedging of CAT derivatives via the most basic operation of an insurance company, i.e. the choice of a suitable risk loading for a particular risk. Future work may combine this approach with other hedging methods, like trading in shares that are correlated with catastrophic events, such as those of construction companies.

The paper is organized as follows: In Section 2 we give the problem description: We assume a global claims process C, which keeps track of all claims due to a specific event in a given country and we consider an insurance company in the same country, so that the index will contain the losses of that particular insurance company among others. The insurance company is facing a certain demand curve which determines the fraction of the insurance market that the company gets to insure, dependent on the risk loading it charges. We therefore have to model the ξ-fraction of the claims process C, for an insurance company with a ξ-fraction of the market. Such a model is constructed in Section 2 where we also derive the wealth process for the insurance company. We conclude that section by giving a short introduction into the concept of utility indifference pricing.

Section 3 constitutes the main part of our paper: We derive a suitable Hamilton–Jacobi–Bellman (HJB) equation for an insurance that holds k units of a derivative written on the total number of claims (Section 3.1). We use the concept of *piecewise deterministic Markov decision process* as presented, e.g., by Bäuerle and Rieder in [16,17]. In particular, we will make use of a verification theorem from [17] to show that a solution to the HJB equation also solves the optimal control problem. At that stage we will have to specialize to exponential utility.

Two Sections 3.3 and 3.4, are devoted to special demand functions. While Section 3.3 looks into the details of the very special case of linear demand, Section 3.4 presents a class of demands that are more general than the previous linear one, but still preserve the property of leading to a unique optimal risk loading. By the end of Section 3, in Section 3.5, we give a numerical example for linear demand. In the technical Section 3.6 we show that the conditions of the verification theorem are satisfied.

Section 4 is devoted to the question under which conditions the derivative could actually be sold, that is, when the buyer's price is at least as big as the seller's price. To that end we study a couple of different pricing concepts related to the utility indifference price.