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A high order finite element scheme for pricing options under regime switching jump diffusion processes

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ABSTRACT

This paper considers the numerical pricing of European, American and Butterfly options whose asset price dynamics follow the regime switching jump diffusion process. In an incomplete market structure and using the no-arbitrage pricing principle, the option pricing problem under the jump modulated regime switching process is formulated as a set of coupled partial integro-differential equations describing different states of a Markov chain. We develop efficient numerical algorithms to approximate the spatial terms of the option pricing equations using linear and quadratic basis polynomial approximations and solve the resulting initial value problem using exponential time integration. Various numerical examples are given to demonstrate the superiority of our computational scheme with higher level of accuracy and faster convergence compared to existing methods for pricing options under the regime switching model.

1. Introduction

1.1. Motivation

In quantitative finance, the seminal works by Black and Scholes [1] and Merton [2] have introduced a performance-free option pricing formula, which does not involve any investor's risk preference or/and subjective views. Ineluctably, the computational simplicity of the derived compact formula has given it a great popularity in the financial sector. The real economy, however, is occasionally disrupted by structural breaks which generate dramatic transitions in market fundamentals causing the macro-economy and financial markets to switch between distinct recurrent regimes. As a result, there have been several studies conducted by market practitioners and academicians on developing models with the ability to efficiently interpret the economic cycles and the changes in the financial time series data due to the regime shifts.

From the stochastic modelling point of view, exponential Lévy processes of finite and infinite activities have been widely applied to describe the salient distributional and stylized behaviours of skewness and kurtosis in financial asset returns, see for instance [3–6] and the numerous references therein. Moreover, many research have also successfully been carried out to explain the smile phenomenon by modelling volatility of the underlying asset price by a stochastic process, see for instance [7]. Additionally, originally proposed in [8] for calibrating business phases of expansions and recessions, the regime switching model characterized by a hidden Markov process has become popular in the recent years for modelling

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the variations in the evolution of asset prices influenced by different macro-economic factors, see also [9]. Indeed, under the Markov switching models, the market parameters depend on states (or regimes) that are determined by an unobserved Markov chain.

Thus, as it will be clearer in the next subsection, the increasing popularity of the regime switching process in option pricing theory is explained mainly due to the inadequacy of the stationary Lévy process to capture cycles of low, moderate and high volatility regimes prevalent in financial markets, which are more appropriately modelled by a Markov process. Further to depicting the jumps in asset prices and the leptokurtic features, a more appealing feature of the regime switching process making it well-known is its ability to model non-linear stylized dynamics of asset returns.

In the next subsection, a brief literature review is presented for the regime switching jump diffusion models, which is the main drive for the numerical pricing of European and American options in the present paper.

1.2. Literature review—new approach

In the literature of financial mathematics, there are many interesting papers that model option pricing under the regime-switching settings. Starting from Naik [10] back in 1993, for the very first time according to our knowledge, the pricing of the European option under a regime-switching model with two regimes has been introduced, and this work has been further extended by several other authors.

Under the regime switching model driven by a hidden Markov process, the market is incomplete and thus, a non-unique equivalent martingale measure can be obtained using the Esscher transform technique, see [11]. Among the recent papers on option pricing problems with the underlying asset price dynamics following a Markovian regime switching process, a closed form solution is derived for the perpetual American option in [12] and lattice methods have been employed in [13–15]. In [9,16], the authors solve a system of partial differential equations of the governing option pricing problem under the regime switching model, where each partial differential equation represents a regime of the underlying economy. Novel numerical algorithms based on the combination of the θ-scheme and explicit treatment of penalty and regime coupling terms are presented in [17] for the systems of free boundary value problems for pricing American options. In option valuation problems, explicit schemes solving the pricing equations are often subject to stability issues and time-step restrictions. In [18], the authors considered the unconditionally stable Crank–Nicolson time-stepping with fixed point iteration to solve a system of nonlinear algebraic equations, where the regime coupling and penalty terms are treated in an implicit manner. That paper, [18], deals with the pricing of options under the Markov-modulated regime switching process by solving the option pricing problem which is posed as a system of coupled partial differential equations (PDEs).

Several numerical methods have been proposed in the literature to find a fair price of an option under the regime switching jump diffusion model. In [19], the authors use a Fourier Space Time-stepping (FST) procedure for pricing path-dependent options by solving a system of transformed partial integro differential equations (PIDEs) under three states Merton jump diffusion model and achieve second order accurate solutions. A similar approximation technique based on Fourier transform is also proposed in [20]. Bastani et al. [21] price American options by considering a mesh-free approach based on a collocation scheme with radial basis functions combined with the implicit Euler time stepping and the resulting numerical scheme produces super-linear and linear rates of convergence in space and time, respectively. Second order accurate numerical schemes under the regime switching jump diffusion models are derived in [22]. Lee [22] solves the American option linear complementarity problem under regime switching Merton [2] and Kou [23] models by using the finite difference method with three level implicit time stepping coupled with an operator splitting approach to yield second order accuracy with respect to the discrete $\mathcal{L}_2$ norm.

The lattice methods (see [15]) are generally simple to implement under the one-dimensional option pricing framework. However, similar to some finite difference based methods, these schemes are prone to stability problems, yield solutions with low level of accuracy and tend to be computationally demanding for higher dimensional complex path-dependent and multi-asset problems. Furthermore, it is also known that the discontinuity in the payoff function adversely affects the rate of convergence of high order finite difference methods which requires complicated co-ordinate transformations to recover the high order convergence rates often at the cost of increased CPU computational times.

Thus, the numerical schemes proposed in the literature so far achieve at most second order rates of convergence under the regime switching jump diffusion model. The new algorithm developed in the present paper for the numerical pricing of European, American and Butterfly options whose asset price dynamics follow the regime switching jump diffusion process differs from the existing computational schemes. Actually, it considers high order Galerkin finite element in a method of lines approach to initially approximate the spatial terms of the system of partial integro differential equations (PIDEs) and the generated initial value problem is then integrated using the exponential time integration. The finite element method deals with a variational integral formulation of the PIDE and thus, the non-differentiability of the payoff function at the point it forms kink does not hinder the level of accuracy for solving option pricing problems as seen in [24,25]. As, in [24], to the best of our knowledge, in the present paper, fourth order convergence rates for American options are achieved for the very first time under the quadratic basis functions in the related quantitative finance literature. Finally, we carry out numerous experiments under two and three states regime switching models to show that highly accurate solutions are achieved under both European and American options, where the early exercise feature of the American option is solved by combining the operator splitting mechanism in the exponential time integration. For the data set in [22], our proposed numerical algorithm achieves an accuracy of 10^{-5} using 256 quadratic finite elements and gives an American option value of 13.8313990 whereby