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A line search exact penalty method with bi-object strategy for nonlinear constrained optimization

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HIGHLIGHTS

- Penalty factor is only related to the information at the current iterate point.
- The sequence of the penalty parameter is non-monotone.
- The search direction is related to the penalty factor.
- The acceptable criterion is not related to the penalty factor.
- Method can handle degenerate problems and inconsistent constraint linearizations.

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ABSTRACT

The exact penalty methods are very popular because of their ability to handle degenerate problems and inconsistent constraint linearizations. This paper presents a line search exact penalty method with bi-object strategy (LSBO) for nonlinear constrained optimization. In the algorithm LSBO, the penalty parameter is selected at every iteration such that the sufficient progress toward feasibility and optimality is guaranteed along the search direction. In contrast with classical exact penalization approaches, LSBO method has two goals to determine whether the current iteration is successful or not. One is improving the feasibility and the other is reducing the value of the objective function. Moreover, the penalty parameter is only related to the information at the current iterate point. The sequence of the penalty parameter is non-monotone, which does not affect the global convergence in theory and is seen to be advantageous in practice. It is shown that the algorithm enjoys favorable global convergence properties under the weaker assumptions. Numerical experiments illustrate the behavior of the algorithm on various difficult situations.

1. Introduction

In this paper, we develop a line search exact penalty method with two-object strategy for finding a local solution of the following nonlinear programming problem

$$\begin{aligned} \min \quad & f(x), \\ \text{s.t.} \quad & c_i(x) \geq 0, \; i \in \mathcal{I} = \{1, \ldots, m\}, \end{aligned} \tag{1.1}$$

where we assume $f : R^n \to R$ and $c_i : R^n \to R^m$ $(i \in \mathcal{I})$ are twice continuously differentiable.

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There are many practical methods for solving problem (1.1), for example, the sequential quadratic programming (SQP) methods [1], the trust-region SQP methods [2], the interior point methods [3] and so on. All optimization methods generate a sequence of trial steps, which are computed as solutions of some quadratic or linear–quadratic model. The criteria for accepting or rejecting trial steps are the strategy for guaranteeing global convergence. One of the strategies is to use some merit function or some penalty function to measure the quality of a trial step. The penalty function is usually a linear combination of the objective function and some measure of constraint violation, where the objective function minimization and the constraint satisfaction are treated together within the framework of a single penalty function minimization problem.

The main difficulty associated with the use of penalty functions is the choice of the penalty parameter. There is usually a threshold value below which the penalty function does not have a local minimum at the solution to (1.1). This threshold value is unknown in advance. If the initial choice of the penalty parameter is too small, the iterates may move away from the solution which may result in an infeasible point of (1.1) or even an unbounded below in the penalty function. On the other hand, if the penalty parameter is excessively large, the penalty function may be difficult to minimize, as emphasizing constraint feasibility too much may lead to small trial steps (or even the rejection of good trial steps) on the curved boundary of the feasible region of (1.1).

In order to handle the selection of the penalty parameter, some researchers present the various techniques without the penalty function, which are called the penalty-free-type methods, for example, see [4–9] and the references therein. Some other people research the updating strategies of the penalty parameter adaptively, e.g., [10–19]. Among those penalty-type methods which use any penalty function, the exact penalty methods are very popular because of their ability to handle degenerate problems and inconsistent constraint linearizations [11,12]. Exact penalty methods have also been used successfully to solve mathematical programs with complementarity constraints (MPCCs) [17], a class of problems that do not satisfy the Mangasarian–Fromovitz constraint qualification at any feasible point.

The idea in this paper is motivated by the penalty-type methods and the penalty-free-type methods. The trial step (the search direction) is computed by a piecewise quadratic model of the exact penalty function. The penalty parameter is selected at every iteration so that the sufficient progress toward feasibility and optimality is guaranteed along the search direction, which is called steering rules [11]. This requires that an auxiliary subproblem (a linear program) be solved in certain cases. Byrd et al. [11] present a line search exact penalty method using steering rules, which requires that the penalty function is descent sufficiently along the search direction at every iteration. However, the method presented here is very different from Byrd's method. The new method has two goals to determine whether the current iteration is successful or not. One is improving the feasibility and the other is reducing the value of the objective function. The new method allows for a certain amount of non-monotonicity on the objective function and on the measure of constraint violation compared to a penalty function approach. Gould et al. [16] present a filter method for nonlinear optimization, where every trial step is computed from subproblems that value reducing both the constraint violation and the objective function. The new method does not use filter technique and its search direction is similar to Byrd et al. [11] but is different from Gould et al. [16]. Moreover, the penalty parameter is only related to the information at the current iterate point. The sequence of the penalty parameter is non-monotone. This property does not affect the global convergence in theory and is seen to be advantageous in practice. Byrd et al. [12] point out clearly that many of the failures caused by large values of the penalty parameter seemed to occur because, near the feasible region, there are often small increases in infeasibility due to nonlinearities in constraints or roundoff error in even linear constraints. Because of the large value of the penalty parameter, these increases dominated the objective function improvement, and forced the penalty method to take very small steps, and sometimes completely prevented further progress. To restrict using excessively large penalty parameter is valuable in practice, which is also confirmed by the numerical examples in this paper.

This paper is divided into four sections. The next section describes the new algorithm. The well definedness of the algorithm is analyzed in Section 3. In Section 4, we analyze the global convergence under the weaker assumptions. Finally, numerical experiments are reported.

2. Description of algorithm

The new algorithm divides two parts. The first part will determine a search direction and the other part will provide a strategy which judge the iteration to be successful or not. An appropriate choice of the penalty parameter will guarantee that the search direction can improve the objective function or the measure of constraint violation. The choice of penalty parameter and the determination of the search direction are similar to Byrd et al. [12]. For the convenience, we simply describe it.

Consider the following unconstrained optimization with l_1 exact penalty function

$$\min_{x\in R^n} P(x,\sigma) = f(x) + \sigma v(x), \tag{2.1}$$

where, $\sigma > 0$ is a penalty parameter and

$$v(x) = \sum_{i\in\mathcal{I}} [c_i(x)]^-,$$

where $[c_i(x)]^- = \max\{0, -c_i(x)\}$. $v(x)$ means the measure of constraint violation.