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Bayes estimation and expected termination time for the competing risks model from Gompertz distribution under progressively hybrid censoring with binomial removals

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ABSTRACT

This paper considers the Bayes estimation for competing risks model under Type-I progressively hybrid censoring with binomial removals from two-parameter Gompertz distribution. Bayes procedure is used to derive the estimations of the unknown parameters, reliability and hazard rate functions based on symmetric and asymmetric loss functions. The effect of varying removal probability p on the expected termination time point under Type-I PHC with binomial removals over the expected termination time point for the complete sample (*REET*) is investigated. Numerical experiments using Monte Carlo simulation are given to compare the performance of the proposed estimates. Finally, one data set was used for illustrative purposes in conclusion.

1. Introduction

Competing risks data occurs in many fields, such as engineering, biological, social science or medical statistics. This competing risks model involves multiple failure modes when only the smallest failure time and the associated failure mode are observed. In reliability analysis, it is common that a failure is associated with one of the several competing failure modes. Since it is not usually possible to study the test units with an isolated competing failure mode, it becomes necessary to assess each failure mode in the presence of other modes. In such a competing risks model, we use the traditional approach to competing risks via latent failure times as suggested by Cox [1]. The failure modes may be assumed to be independent so as to avoid the problem of model identifiability. There are several papers in competing risks modeling assuming independence among competing failure modes and the competing failures data follow different lifetime distributions (see, e.g., [2]). Mao et al. [3] studied the competing risks model from exponential distribution. Mazucheli et al. [4] considered the case when the competing risks have a Lindley distribution. Sarhan [5] analyzed the competing risks models with generalized exponential distributions. Other related works see, [6,7].

In addition to multiple failure modes, due to time constraint and cost reduction, it is not possible to observe enough samples in an experiment, so censoring is inevitable in life testing. Under Type-II progressively censoring scheme, Hussaini et al. [8] studied the competing risks model from half- logistic distributions. Cramer et al. [9] presented a competing risks model based on Lomax distributions. Pareek et al. [10] studied the same latent failure time model from Weibull distribution.

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Kundu et al. [11] considered the analysis of competing risks data from exponential distribution under the latent failure time model. Under other censoring schemes, such as generalized Type-I or Type-II hybrid censoring, Mao et al. [3] and Chandrasekar et al. [12] presented exact likelihood inference with exponential competing risks model.

In this paper, we present a competing risks model under Type-I progressively hybrid censoring scheme from Gompertz distribution. The Gompertz distribution is one of classical mathematical models and was first introduced by Gompertz [13], which is a commonly used growth model in actuarial and reliability and life testing, and plays an important role in modeling human mortality and fitting actuarial tables and tumor growth. This distribution has been widely used, the related works see, [14,15].

The Type-I progressively hybrid censoring (PHC) was first proposed by Kundu et al. [16] (see also [17]). As the name suggests, it is a mixture of Type- I progressively and hybrid censoring schemes. This censoring scheme has become quite popular for analyzing highly reliable data and has been widely used in reliability analysis, see, [18–20]. This censoring scheme can be defined as follows: suppose n identical units are put to life test with progressive censoring scheme $(r_1, r_2, \dots, r_r)$, $1 \le r \le n$, the experiment is terminated at time τ, where, $\tau \in (0, \infty)$, r_i $(i = 1, \dots, r)$ and r are fixed in advance. At the time of the first failure t_1, r_1 of the remaining units are randomly removed, at the time of the second failure t_2, r_2 of the remaining units are randomly removed and so on. If the rth failure time t_r occurs before time τ, all the remaining units $r_r = n - r - (r_1 + \cdots + r_{r-1})$ are removed and the terminal time of the experiment is t_r. On the other hand, if the rth failure time t_r does not occur before time τ and only J failures occur before time τ, where, $0 \le J \le r$. Then at the time τ, all the remaining r_τ^* units are removed, where $r_\tau^* = n - J - (r_1 + \cdots + r_J)$, and the terminal time of the experiment is τ. We denote the two cases as Case 1 and Case 2.

Case 1: $t_1 < t_2 < \cdots < t_r$, if $t_r < \tau$.

Case 2: $t_1 < t_2 < \cdots < t_J < \tau < t_{J+1} < \cdots < t_r$, if $t_r > \tau$.

The main focus of this paper is the analysis of the competing failure data from Gompertz distribution under Type-I PHC with binomial removals. The rest of the paper is organized as follows. In Section 2, we provide the competing risks model. Bayes estimations based on symmetric and asymmetric loss functions are presented in Section 3. The expected termination time under Type-I PHC scheme with binomial removals is discussed in Section 4. Section 5 provides the numerical experiments and data analysis.

2. Competing risks model

The approach discussed in this paper is based on the following two assumptions.

A1. There are m independent competing failure modes, the failure of a system occurs only due to one of the m competing failure modes with lifetimes $T_1, \dots, T_m$, and the failure time of the system is $\min\{T_1, \dots, T_m\}$.

A2. The lifetime of competing failure mode j $(j = 1, \dots, m)$, denoted by T_j, follows a Gompertz distribution with shape parameter λ_j and scale parameter θ_j. The cumulative distribution function (CDF) and the probability density function (PDF) are given, respectively, as

$$F_j(t|\lambda_j, \theta_j) = 1 - \exp\{-(\theta_j/\lambda_j)(e^{\lambda_j t} - 1)\}, \quad t > 0,\ \lambda_j > 0,\ \theta_j > 0, \tag{1}$$

$$f_j(t|\lambda_j, \theta_j) = \theta_j e^{\lambda_j t} \exp\{-(\theta_j/\lambda_j)(e^{\lambda_j t} - 1)\}, \quad t > 0,\ \lambda_j > 0,\ \theta_j > 0, \tag{2}$$

and the reliability function and hazard rate function are given as

$$S_j(t) = \exp\{-(\theta_j/\lambda_j)(e^{\lambda_j t} - 1)\}, \quad t > 0,\ \lambda_j > 0,\ \theta_j > 0,$$
$$H_j(t) = \theta_j e^{\lambda_j t}, \quad t > 0,\ \lambda_j > 0,\ \theta_j > 0.$$

Under Type-I PHC, let τ^* denote the terminal time of the experiment, r^* denote the number of failures before time τ^*, where $\tau^* = \min\{t_r, \tau\}$, and $r^* = r$, if $t_r \le \tau$, $r^* = J$, if $t_r > \tau$. $(t_1, \alpha_1), \dots, (t_{r^*}, \alpha_{r^*})$ are the observed failure data, where $t_1, t_2, \dots, t_{r^*}$ are order statistics denoting the failure time, α_i takes any integer in the set $\{1, 2, \dots, m\}$. $\alpha_i = j$ $(j = 1, \dots, m)$ indicates the failure is caused by the jth failure mode. Let $\delta_j(\alpha_i) = 1$, if $\alpha_i = j$, $\delta_j(\alpha_i) = 0$, if $\alpha_i \ne j$, $n_j = \sum_{i=1}^{r^*} \delta_j(\alpha_i) \ge 0$ denote the number of failures caused by the jth failure mode. We reasonably assume that $r_r = 0$ for Case 1 and there is at least one failure caused by the jth failure mode for the two cases, the likelihood function for the two cases can be written as

$$L_j(t|\lambda_j, \theta_j, r.) \propto \prod_{i=1}^{r^*} f_j(t_i)^{\delta_j(\alpha_i)} [1 - F_j(t_i)]^{1-\delta_j(\alpha_i)} [1 - F_j(t_i)]^{r_i} [1 - F_j(\tau^*)]^{n - r^* - \sum_{i=1}^{r^*} r_i}$$
$$\propto \theta_j^{n_j} \exp\left\{\lambda_j \sum_{i=1}^{r^*} \delta_j(\alpha_i) t_i - (\theta_j/\lambda_j)\left[\sum_{i=1}^{r^*} (r_i + 1)(e^{\lambda_j t_i} - 1) + \left(n - r^* - \sum_{i=1}^{r^*} r_i\right)(e^{\lambda_j \tau^*} - 1)\right]\right\}.$$