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The mean wasted life time of a component of system

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ABSTRACT

A reliability inspection model in which a component of a technical system has lifetime X and inspection time S is considered. It is assumed that X and S are random variables with absolutely continuous joint distribution function $F_{X,S}$, and the joint probability density function $f_{X,S}$. Firstly, we consider mean residual life function of the component under two different setups of inspection. Secondly, we consider an inspection model where at the inspection time the component is replaced with its spare regardless of whether the component is alive or failed at this time. Under condition that $0 < t < S < X$ we are interested in expected value of $X - S$, which is the mean wasted time of intact at time t component in the case if it will not be failed at inspection time, but will be replaced with the new one. We derive formula for mean wasted life time expressed in terms of $f_{X,S}$ and partial derivatives of $F_{X,S}$. Some examples with graphical representations are also provided.

1. Introduction

Let X be a life length of any item, which may be a live organism, a technical system or a component of a system. In this paper, for clarity, we assume that this item is a component of a technical system. Furthermore, we assume that X is an absolutely continuous random variable with distribution function (cdf) $F_X(x)$ and probability density function (pdf) $f_X(x)$. Let $\omega(F) = \sup\{u : F(u) < 1\}, \alpha(F) = \inf\{u : F(u) > 0\} = 0$ be the right and left endpoints of the support of F and assume that $\alpha(F) = 0$. One of the important concepts in reliability engineering is the mean residual life function. The function $\psi_F(t) = E(X - t \mid X > t)$ expresses the average remaining lifetime of the component which is intact at time t, and it is called the mean residual life (MRL) function of the component. The concept of mean residual life function aroused the interest of reliability engineers as well as many researchers working on statistical theory of reliability. Among the first papers dealing with the MRL function is the paper by Hall and Wellner [1], who considered the class of distributions with linear mean residual life function, i.e. $\psi_F(t) = \alpha t + \beta$, for $\alpha > -1$ and $\beta > 0$. It is interesting to point out that the MRL function, similar to the hazard rate function, characterizes the distribution function and an inverse formula

$$F_X(x) = 1 - \frac{\psi_F(0)}{\psi_F(x)} \exp\left\{-\int_0^x \frac{dt}{\psi_F(t)}\right\}, \quad t < \omega(F),$$

allows to determine the distribution function if we know the MRL function. Oakes and Dasu [2] characterized a family of survival distributions in terms of the residual life distributions and linear MRL functions. More developments on the MRL functions are due to extension of this concept to the system level. In this context, Bairamov et al. [3] first considered a parallel system of n components with corresponding lifetimes $X_1, X_2, \ldots, X_n$ and introduced an MRL function of this system as

$$\Psi_n(t) = E(X_{(n)} - t \mid X_{(1)} > t),$$

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where $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ are order statistics of $X_1, X_2, \ldots, X_n$. The function $\Psi_n(t)$ can be interpreted as the remaining life of the parallel system given that none of the components has failed at the time t. This MRL function involves order statistics and deals with the special case of general coherent systems. Afterwards, many researchers have considered an extension of this MRL function, for example, Asadi and Bayramoglu [4,5] considered extension to k-out-of-n system under condition that $m < k$ of the components are intact at time t. There are numerous papers that appeared in statistical literature last years considering MRL functions of different type of systems including the coherent system. For a comprehensive description of related works investigating the MRL functions under different conditions on dependency between components and extension to more complex systems with two or more dependent components per element we refer [6]. Recently, Tavangar and Bairamov [7] studied the conditional residual lifetime of coherent system, under different monitoring, more precisely under condition that at a fixed time some of the components have failed but still there are functioning components. The works connected with the MRL functions paved a way to a new idea, to consider the mean remaining strength of a coherent system in the stress–strength setup. Bairamov et al. [8] and Gurler et al. [9] have considered the mean residual stress for a k-out-of-n system with exchangeable components in the stress–strength setup. The mean remaining stress is important in reliability and mechanical engineering.

In this paper we consider a different type of a reliability model. We assume that the work and control of a system is organized such a way that there is an inspection time to control and step into work of the system. In related models used in survival analysis and medicine this may be interpreted as an inspection time of a live organism or control time of AIDS patients.

Considering a component of a technical system we assume that X is a lifetime of a component as described above. Furthermore, as we pointed above, we assume that for this system there is an inspection time S, which is also a random variable with absolutely continuous distribution with cdf $F_S(x)$ and pdf $f_S(x)$. We assume that X and S are stochastically dependent random variables, i.e. the inspection time in general depends on the lifetime of the component. It is reasonable, for example to use positive quadrant dependent bivariate distribution for modeling of the joint distribution function of (X, S), i.e. the increase of the lifetime X is resulted in increased inspection time. We consider two models, both having practical applications in system reliability, survival analysis and medicine. In the first model we assume that if at inspection time the component is alive we do not interfere with the system, it continues to work and the recorded lifetime of the component is assumed to be X. Otherwise, the component is changed with the new one and for future projection it is assumed that the recorded lifetime of the component is S.

In the second model, at inspection time the component is changed with the new one, even though, if it is not failed at this time. In the second model we introduce a new concept, Mean Wasted Life Time (MWLT) function, which is the average value of wasted time in the case when we change the component at inspection time while it is not failed at this time and would be working perfectly until it fails. This function is important, because it helps in the planning of inspection times for the system. In Section 2 we consider the mean residual life functions for both models. The main results of the paper are given in Section 3, where we introduce the MWLT function, and present results concerning this function. Examples and graphs are also provided in Section 3.

2. Mean residual life functions with inspection

2.1. Model 1

At time S the system is inspected and if the component is alive at this time, then it continues to function and its technical lifetime is assumed to be X. If at inspection time it turns to be that the component has been failed, then its lifetime is fixed as S, i.e. it is assumed that the technical lifetime of the component of considered system is S. In this case, we assume that failure time of a component is unknown, we have just an information that at inspection time it was failed. Therefore, if T denotes the technical lifetime of the component, that is used in a technical process, then

$$T = \max(X, S).$$

For example, assume that a periodic maintenance and repair service of the lighting system of a company is determined as one month. The component we are considering is a certain lamp with lifetime X. If the lamp is alive at the periodic service time, then it continues to function and its lifetime is considered as X. If the lamp appears to be failed before service time, then its lifetime is accepted to be one month and this information is used in planning the service works.

We assume that X and S are stochastically dependent random variables with joint cdf $F_{X,S}(x, s)$ and marginal cdf's are $F_X(x) = F_{X,S}(x, \infty)$ and $F_S(s) = F_{X,S}(\infty, s)$. The mean residual life (MRL) function of inspected item is

$$\Psi_1(t) = E(T - t \mid T > t), \tag{1}$$

and the cdf of conditional random variable $(T - t \mid T > t)$ is

$$\begin{aligned} F_{(T-t|T>t)}(x) &= P\{T - t \leq x \mid T > t\} \\ &= \frac{F_{X,S}(t + x, t + x) - F_{X,S}(t, t)}{1 - F(t, t)}. \end{aligned}$$