Journal of Computational and Applied Mathematics 306 (2016) 40–52

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Optimal insurance risk control with multiple reinsurers

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ARTICLE INFO

Article history:
Received 7 April 2015

MSC:
primary 93E20
secondary 60J25
91B30

Keywords:
Variance premium principle
Reinsurance strategy
Multiple reinsurers
Fixed costs
Taxes
HJB equation

ABSTRACT

An optimal insurance risk control problem is discussed in a general situation where several reinsurance companies enter into a reinsurance treaty with an insurance company. These reinsurance companies adopt variance premium principles with different parameters. Dividends with fixed costs and taxes are paid to shareholders of the insurance company. Under certain conditions, a combined proportional reinsurance treaty is shown to be optimal in a class of plausible reinsurance treaties. Within the class of combined proportional reinsurance strategies, analytical expressions for the value function and the optimal strategies are obtained.

1. Introduction

Reinsurance is one of the practical means adopted by insurance companies to transfer insurance risk. It provides a way to control risk, and so, to enhance the financial stability of an insurance company. From both the theoretical and practical perspectives, it may be interesting to discuss what is an optimal level of reinsurance that an insurance company should acquire. This problem is widely known as an optimal reinsurance problem in the risk theory literature. It has also captured some attention of many academic researchers in actuarial science and insurance. This might be partly attributed to the intellectual challenge of the problem. A popular approach to study an optimal reinsurance problem is to use stochastic optimal control theory in continuous-time to discuss the optimization problem. Some works in this direction are, for example, [1–12], amongst others. Two major types of reinsurance strategies such as the excess of loss reinsurance strategy and the proportional reinsurance strategy have been the main focuses of these works. Furthermore, for the sake of mathematical convenience, the expected value premium principle is widely used for calculating premiums in the literature on optimal reinsurance. However, the use of expected value premium principle may be questioned on both theoretical and practical grounds. From the theoretical point of view, the expected value premium principle cannot incorporate the volatility of claims losses which describes fluctuations in claims losses and can be measured by the standard deviation or variance of the claims losses. From the practical perspective, it has been noted in [13] that "*In insurance practice, the most widely used method is to base calculation on the first two moments*". The variance premium principle can capture the first two moments

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http://dx.doi.org/10.1016/j.cam.2016.04.005
0377-0427/

in premium calculations. It has been used by some authors to investigate an optimal insurance risk control problem, see, for example, [14,15,4,16,17,10,11].

Much attention has been paid to the situation where an insurer transfers the risk exposure to only one reinsurer. Relatively little attention has been given to a general situation that multiple reinsurers participate in a reinsurance treaty. Under the criterion of minimizing value at risk (VaR) or conditional value at risk (CVaR) of an insurer's total risk exposure, Chi and Meng [18] studied an optimal reinsurance arrangement in the presence of two reinsurers, where the first reinsurer adopts the expected value premium principle while the second reinsurer uses the premium principle satisfying three axioms: distributional invariance, risk loading and preserving stop-loss order. Asimit et al. [19] also supposed that an insurance company may be able to share the risk with two reinsurers, where the first reinsurer uses the expected value premium principle and the second reinsurer adopts a distorted premium principle. The two papers refer to a static, single-period, insurance risk model. In a continuous-time set up, Meng [17] studied an optimal risk control problem with two reinsurers who calculate premiums by the variance premium principle with different parameters. Meng et al. [10] also considered an optimal reinsurance problem with two reinsurers in a continuous-time set up, where the two reinsurers adopted an expected value premium principle and a variance premium principle, and the optimization criterion was the probability of ruin. It seems that little attention has been given to study an optimal reinsurance problem with more than two reinsurers in a continuous-time set up. For the sake of generality, it may be of interest to consider the situation where more than two reinsurers participate in the reinsurance treaty. In a continuous-time set up, an optimal reinsurance problem with more than two reinsurers is of theoretical interest and intellectual challenge since it is a high-dimensional stochastic optimal control problem.

In this paper, an optimal insurance risk control problem is studied in a general situation where several reinsurance companies enter into a reinsurance treaty with an insurance company. These reinsurance companies adopt variance premium principles with different parameters. In addition to determining an optimal reinsurance level, another key problem for an insurance company is to determine an optimal level of dividend payments to its shareholders. This is known as an optimal dividend problem. De Finetti [20] pioneered a formal study of an optimal dividend problem, where the expected present value of all dividends before possible ruin was maximized. The seminal work of De Finetti [20] has stimulated a lot of interest among researchers in actuarial science. A combination of an optimal reinsurance problem with multiple reinsurers and an optimal dividend problem is discussed, where dividends with fixed costs and taxes are paid to shareholders of the insurance company. Mathematically, the optimal dividend problem is related to an impulse control problem and has been studied in the literature, see for example, [21,8,5–7,17,9,22,11]. Under certain conditions, a combined proportional reinsurance treaty is shown to be optimal in the class of plausible reinsurance treaties. Within the class of combined proportional reinsurance strategies, analytical expressions for the value function and the optimal strategies are provided.

The rest of the paper is organized as follows. In the next section, the dynamic risk control problem with m reinsurers with variance premium principles is formulated. The corresponding optimization problem is presented. In Section 3, the optimality of a combined proportional reinsurance strategy is discussed. In Section 4, analytical expressions for the value function, the optimal reinsurance and dividend strategy are derived. The final section summarizes the paper.

2. Model formulation

Uncertainty is resolved over time according to a complete, filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$, where the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies the usual conditions (i.e., the right continuity and $\mathbb{P}$-completeness) and $\mathbb{P}$ is a real-world probability measure. According to the Cramér–Lundberg model, the surplus of an insurance company is given by:

$$P(t) = x + pt - \sum_{i=1}^{N(t)} Z_i,$$

where $x \geq 0$ is the initial surplus; $\{N_t, t \geq 0\}$ is a Poisson process with constant intensity parameter $\lambda > 0$; The claims $Z_i,\ i = 1, 2, \ldots$ are independent and identically distributed (*i.i.d.*) random variables, and are independent of $\{N_t, t \geq 0\}$. Assume, for each $i = 1, 2, \ldots, Z_i$ has a finite mean μ and a finite second moment σ^2. The premium p is determined by the variance premium principle with the parameter $\theta_0 > 0$, i.e.,

$$pt = \mathbb{E}\left[\sum_{i=1}^{N(t)} Z_i\right] + \theta_0 \mathbb{D}\left[\sum_{i=1}^{N(t)} Z_i\right] = \lambda(\mu + \theta_0\sigma^2)t,$$

where $\mathbb{E}[\cdot]$ and $\mathbb{D}[\cdot]$ are the expectation and variance operators, respectively. To control its risk exposures, the insurance company can cede part of the loss for each claim by acquiring reinsurance. We assume that m reinsurance companies participate in a reinsurance treaty and these reinsurance companies adopt variance premium principles with different parameters, say $\theta_j,\ j = 1, 2, \ldots, m$. Without loss of generality, we assume that $\theta_i \geq \theta_j, i < j$. For each claim Z_i, the jth reinsurance company undertakes $g_j(Z_i)$. Then $g_0(Z_i) := Z_i - \sum_{j=1}^{m} g_j(Z_i)$ is the remaining part of the claim Z_i, which is retained by the insurance company. The aggregate premium that is received by these m reinsurance companies from the insurance company is given by:

$$\sum_{j=1}^{m}\left\{\mathbb{E}\left[\sum_{i=1}^{N(t)} g_j(Z_i)\right] + \theta_j \mathbb{D}\left[\sum_{i=1}^{N(t)} g_j(Z_i)\right]\right\} = \lambda t \sum_{j=1}^{m}\{\mathbb{E}[g_j(Z_i)] + \theta_j \mathbb{E}[g_j(Z_i)]^2\}.$$