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Asymptotic ruin probability of a renewal risk model with dependent by-claims and stochastic returns

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ABSTRACT

Consider a nonstandard renewal risk model, in which every main claim induces a delayed by-claim. Suppose that the surplus is invested to a portfolio of one risk-free asset and one risky asset, and the main claim sizes with by-claim sizes form a sequence of pairwise quasi-asymptotically independent random variables with dominatedly varying tails. Under this setting, asymptotic behavior of the ruin probability of this renewal risk model is investigated, by establishing a weakly asymptotic formula, as the initial surplus tends to infinity. Some numerical results are also presented to illustrate the accuracy of our asymptotic formulae.

1. Model and introduction

Consider a nonstandard risk model, in which every main claim may be accompanied with another type of claim, called a by-claim. The time of occurrence of a by-claim is later than that of its main claim, and the time of delay for a by-claim is random. This kind of risk model may be of practical use. For instance, a serious motor accident causes different kinds of claims, such as car damage, injury and death; some can be dealt with immediately while others need a random period of time to be settled.

Let the main claim sizes X_k, $k = 1, 2, \ldots$, and the by-claim sizes Y_k, $k = 1, 2, \ldots$, both form a sequence of nonnegative and identically distributed random variables with common distribution functions F and G, respectively. Assume that the main claims arrive according to a renewal counting process $\{N(t); t \geq 0\}$ with independent and identically distributed (i.i.d.) interarrival times θ_k, $k = 1, 2, \ldots$, and the renewal function $\lambda_t = \mathsf{E}N(t)$. Denote by $\tau_i = \sum_{k=1}^{i} \theta_k$, $i = 1, 2, \ldots$, the main claim arrival times of $\{N(t); t \geq 0\}$. Let D_k, $k = 1, 2, \ldots$, be the corresponding delay times of the by-claims, which form a sequence of nonnegative, but possibly degenerated at 0, and identically distributed random variables with common distribution function H.

Suppose that an insurance company commence at time 0 with initial wealth $x \geq 0$, and then the cash flow of premiums less claims is modeled as a compound renewal process with the form

$$C_t = ct - \sum_{i=1}^{N(t)} \Big(X_i - Y_i I(\tau_i + D_i \leq t) \Big), \quad t \geq 0,$$

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where $c \geq 0$ is a fixed rate of premium payment. It is obvious that the incorporation of the aggregate by-claims process complicates the analysis of surplus process, and has been studied by many scholars; see e.g. [4,16,23–25,27,28], among others.

Due to the insurance company is allowed to invest its surplus into a portfolio consisting of risk-free and risky assets, we assume that the insurance company invests its reserve into a bond with a constant interest rate $\delta > 0$, and some stock, modeled by an exponential Lévy process. Their price processes are described as follows:

$$R_0(t) = e^{\delta t} \quad \text{and} \quad R_1(t) = e^{L(t)}, \quad t > 0,$$

with $R_0(0) = 1$ and $R_1(0) = 1$, where $\{L(t); t \geq 0\}$ is a Lévy process which starts with 0, has independent and stationary increments, and is almost surely right continuous with Lévy triplet (γ, σ^2, ρ), where $-\infty < \gamma < \infty, \sigma \geq 0$, are two constants and ρ is a Lévy measure on $(-\infty, \infty)$ satisfying $\rho(\{0\}) = 0$ and $\int_{-\infty}^{\infty}(x^2 \wedge 1)\rho(\mathrm{d}x) < \infty$. For the general theory of Lévy process, see [1,8,20]. Note that it is impossible for an insurance company to put all of its surplus to the risky assets, and thus it is often supposed that the insurance company invests a constant fraction $r \in [0, 1]$ of the surplus in the risky assets and keeps the remaining surplus in the risk-free asset at each point of time. This is the so-called constant investment strategy and commonly used in mathematical finance and actuarial science; see e.g. [10,11,18,19], among many others.

With the constant fraction $r \in [0, 1]$, the portfolio investment return process is defined to be the solution of the following stochastic differential equation:

$$\mathrm{d}R_r(t) = R_r(t-)\mathrm{d}\hat{L}_r(t), \quad t > 0, \qquad R_r(0) = 1, \tag{1.1}$$

where $\mathrm{d}\hat{L}_r(t) = (1-r)\delta\mathrm{d}t + r\mathrm{d}\hat{L}(t)$ and $\hat{L}(t)$ satisfies $\mathrm{d}R_1(t) = R_1(t-)\mathrm{d}\hat{L}(t)$. By Proposition 8.22 in [8], Eq. (1.1) admits the solution

$$R_r(t) = e^{L_r(t)}, \quad t > 0, \qquad R_r(0) = 1,$$

where

$$L_r(t) = \hat{L}_r(t) - \frac{1}{2}[\hat{L}_r, \hat{L}_r]_t + \sum_{0<s\leq t}\left(\log\left(1 + \Delta\hat{L}_r(s)\right) - \Delta\hat{L}_r(s) + \frac{1}{2}\left(\Delta\hat{L}_r(s)\right)^2\right)$$

with $\Delta\hat{L}_r(s) = \hat{L}_r(s) - \hat{L}_r(s-)$ and $[\hat{L}_r, \hat{L}_r]_t$ being the quadratic variation process of $\hat{L}_r$. By Lemma 2.5 of Emmer and Klüppelberg [10], $\{L_r(t); t \geq 0\}$ is also a Lévy process with characteristic triplet $(\gamma_r, \sigma_r^2, \rho_r)$, specified by the original Lévy process $L(t)$ as follows:

$$\gamma_r = \gamma r + (1-r)(\delta + r\sigma^2/2) + \int_{-\infty}^{\infty}\left(\log\left(1 + r(e^x - 1)\right)I(|\log(1 + r(e^x - 1))| \leq 1) - rxI(|x| \leq 1)\right)\rho(\mathrm{d}x),$$

$\sigma_r^2 = r^2\sigma^2$ and $\rho_r(A) = \rho\left(\left\{-\infty < x < \infty : \log\left(1 + r(e^x - 1)\right) \in A\right\}\right)$ for any Borel set $A \subset \mathbb{R}$ (here and in the sequel, $I(A)$ denotes the indicator function of the set A). Define the Laplace exponent of $\{L_r(t); t \geq 0\}$ as

$$\phi_r(z) = \log \mathrm{E}e^{-zL_r(1)}, \quad z \in (-\infty, \infty).$$

If $\phi_r(z)$ is finite, then

$$\mathrm{E}e^{-zL_r(t)} = e^{t\phi_r(z)} < \infty, \quad t \geq 0,$$

and it is easy to verify that $\phi_r(z)$ is convex in z (see [8,20] for details).

Following the method used by Klüppelberg and Kostadinova [15], we define the integrated risk process as the result of the insurance business and the net gains of the investment, that is, the solution to the stochastic differential equation:

$$\mathrm{d}U_r(t) = \mathrm{d}C_t + U_r(t-)\mathrm{d}\hat{L}_r(t), \quad t > 0, \ U_r(0) = x, \tag{1.2}$$

where x is the initial capital. Due to the assumption that the insurance and the investment processes are independent, it follows from Lemma 2.2 of Klüppelberg and Kostadinova [15] that, the solution to (1.2), i.e., the surplus process of the insurance company, is

$$U_r(t) = e^{L_r(t)}\left(x + \int_0^t e^{-L_r(s)}\mathrm{d}C_s\right). \tag{1.3}$$

Particularly, when we take $r = 0$, model (1.3) reduces to

$$U_0(t) = xe^{\delta t} + c\int_0^t e^{\delta(t-s)}\mathrm{d}s - \sum_{i=1}^{N(t)}\left(X_i e^{\delta(t-\tau_i)} - Y_i e^{\delta(t-\tau_i-D_i)}I(\tau_i + D_i \leq t)\right), \tag{1.4}$$

which is called by-claim risk model with a constant interest rate.