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A computational method for solving nonlinear stochastic Volterra integral equations

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ABSTRACT

In this paper, an efficient method for solving nonlinear stochastic Itô–Volterra integral equations (NSIVIEs) is proposed. By using new adjustment of hat basis functions and their stochastic operational matrix of integration, the NSIVIE is reduced to a nonlinear system of algebraic equations. Also, the error analysis of the proposed method is investigated and proved that the order of convergence is $O(h^4)$. Finally, numerical examples affirm the efficiency of the proposed method.

1. Introduction

In recent years, many different methods and different basis functions, such as homotopy perturbation method [1,2], radial basis functions method [3,4], transform methods [5], block pulse functions method [6–8], wavelets methods [9], Adomian decomposition method [10–14], Nystrom's method [15,16], Taylor expansion method [17,18], block-pulse functions and Taylor series method [19,20], triangular functions method [21] and hat functions method [22], were applied to derive solutions of different kinds of integral equations.

Babolian and Mordad [23] have applied hat functions to obtain approximate solutions of linear and nonlinear integral equations. The basic idea in this paper is developing a new numerical method for solving NSIVIEs, which is based on adjustment of hat basis functions. Consider NSIVIEs as

$$x(t) = f(t) + \int_0^t k_1(t,s)\mu(s,x(s))ds + \int_0^t k_2(t,s)\varphi(s,x(s))dB(s), \quad 0 \le t \le T, \tag{1}$$

where $f(t)$, $k_1(t,s)$ and $k_2(t,s)$, for $t, s \in [0, T]$, are the known functions on the probability space, $x(t)$ is the unknown random function, $B(t)$ is a standard Brownian motion and $\int_0^t k_2(t,s)\varphi(s,x(s))dB(s)$ is the Itô integral. Also, assume that $\mu(t,x(t))$ and $\varphi(t,x(t))$ are analytic functions.

For easy reference, definitions of adjustment of hat basis functions and their properties are given in Section 2. In Section 3, the stochastic integration operational matrix is given. NSIVIEs are solved by applying the stochastic operational matrix, in Section 4. In Section 5, the error analysis is proved. In Section 6, the proposed method is used for solving some numerical examples. Finally, Section 7 affords some brief conclusion.

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2. Definitions of adjustment of hat basis functions and their properties

A set of adjustment of hat functions are defined on $[0, T]$ as

$$\phi_0(t) = \begin{cases} \dfrac{-1}{6h^3}(t-h)(t-2h)(t-3h) & 0 \le t \le 3h, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

if $i = 3k-2$ and $1 \le k \le \frac{n}{3}$

$$\phi_i(t) = \begin{cases} \dfrac{1}{2h^3}(t-(i-1)h)(t-(i+1)h)(t-(i+2)h) & (i-1)h \le t \le (i+2)h, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

if $i = 3k-4$ and $2 \le k \le \frac{n}{3}+1$

$$\phi_i(t) = \begin{cases} \dfrac{-1}{2h^3}(t-(i-2)h)(t-(i-1)h)(t-(i+1)h) & (i-2)h \le t \le (i+1)h, \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

if $i = 3k$ and $1 \le k \le \frac{n}{3}-1$

$$\phi_i(t) = \begin{cases} \dfrac{1}{6h^3}(t-(i-3)h)(t-(i-2)h)(t-(i-1)h) & (i-3)h \le t \le ih, \\ \dfrac{-1}{6h^3}(t-(i+1)h)(t-(i+2)h)(t-(i+3)h) & ih \le t \le (i+3)h, \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

and

$$\phi_n(t) = \begin{cases} \dfrac{1}{6h^3}(t-(T-h))(t-(T-2h))(t-(T-3h)) & (T-3h) \le t \le T, \\ 0 & \text{otherwise}, \end{cases} \tag{6}$$

where $h = \frac{T}{n}$ and $n \ge 3$ is an integer of multiple three.

Let us divide interval $[0, T]$ into $\frac{n}{3}$ subintervals $[ih, (i+3)h]$ where $i = 0, 3, \ldots, n-3$, of equal lengths $3h$. By using the definition of adjustment hat functions, we have

$$\phi_i(kh) = \begin{cases} 1 & i = k, \\ 0 & i \ne k, \end{cases} \tag{7}$$

and

$$\sum_{i=0}^{n} \phi_i(t) = 1.$$

An arbitrary real function $f(t)$ on $[0, T]$ can be expanded in an adjustment of hat series as follows

$$f(t) \simeq \sum_{i=0}^{n} f_i\phi_i(t) = F^T\Phi(t) = \Phi^T(t)F, \tag{8}$$

where

$$F = [f_0, f_1, \ldots, f_n]^T,$$

and

$$\Phi(t) = [\phi_0(t), \phi_1(t), \ldots, \phi_n(t)]^T, \tag{9}$$

with

$$f_i = f(ih), \quad i = 0, 1, \ldots, n. \tag{10}$$

Also, expand $\int_0^t \Phi(t)dt$ by relation (8) in terms of the adjustment of hat basis functions as

$$\int_0^t \Phi(t)dt \simeq \sum_{j=0}^{n} a_{i,j}\Phi_j(t), \quad i = 0, 1, \ldots, n. \tag{11}$$