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Solving generalized pantograph equations by shifted orthonormal Bernstein polynomials

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ABSTRACT

In this paper, we introduce Shifted Orthonormal Bernstein Polynomials (SOBPs) and derive the operational matrices of integration and delays for these polynomials. Then, we apply them to convert the pantograph equations to a system of linear equations. An important property of this method is that the condition number of the coefficient matrix of the system is small which confirms that our method is stable. Error analysis and comparison with other methods are given to confirm the validity, efficiency and applicability of the proposed method.

1. Introduction

Delay differential equations have a wide range of applications in industrial, biological, chemical, electronic and transportation systems [1–3].

Functional differential equations with proportional delay are usually referred to pantograph equations or generalized pantograph equations [1,2,4–6].

Properties of the analytic solution of these equations and also their numerical solutions have been studied by several authors [7–12]. Moreover, stability properties of some numerical methods for nonlinear generalized pantograph equations are discussed in [13–21]. Using orthogonal functions in the expanding methods have received considerable attention in dealing with various problems [3,22,23]. The main characteristic of these techniques is that it reduces these problems to a system of algebraic equations, thus greatly simplify the problems.

Because of their many useful properties, Bernstein polynomials have significant usage in the aided geometric design and some other fields of mathematics such as numerical solving of partial differential equations [24–28], optimal control theory [29–31] and stochastic dynamics [32].

Despite the many useful features, Bernstein polynomials have not orthogonality property. For this purpose the orthonormal Bernstein polynomial basis can be generated through the Gram–Schmidt orthonormalization process, apart from numerical instability of Gram–Schmidt process, this process must be repeated when the degree of the polynomial basis is increased. This difficulty can be solved by using the explicit formula of orthonormal Bernstein polynomials which has been derived in [33].

In this paper for shifted orthonormal Bernstein polynomials, we have established operational matrices of integration P and delays Θ, Π, $\Upsilon^{(jk)}$, by a general procedure. These matrices have been used to calculate the solution of the following

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generalized pantograph equation with variable coefficients [23]

$$y^{(r)}(t)=\sum_{j=0}^{\ell}\sum_{k=0}^{r-1}p_{jk}(t)y^{(k)}(\lambda_{jk}t+\mu_{jk})+g(t),\quad t\in[0,L],\tag{1}$$

subject to the initial conditions

$$y^{(i)}(0)=\lambda_i,\quad i=0,\ldots,r-1,\tag{2}$$

where λ_i, λ_{jk} and μ_{jk} are real or complex numbers and p_{jk} and g are continuous functions defined on the interval $[0,L]$.

Bernstein polynomials have already been used to solve pantograph equations [31]. Here, we use the shifted orthonormal Bernstein polynomials and discuss the convergence analysis. Moreover, we obtain a proper upper bound in Theorem 3 for the residual error. The numerical results presented in Section 5 show that these upper bounds are very sharp. In addition, a distinguished feature of using these basis functions (compared with approaches using different basis [34,35,23]) is that the algebraic systems of the present approach have very small condition number.

This paper is organized as follows:

In Section 2, we describe the basic formulation of shifted orthonormal Bernstein polynomials, expansion of SOBPs in terms of Taylor basis, the function approximation and the operational matrices of integration. Section 3 is devoted to calculating the operational matrices of delay operators for solving the pantograph equations. In Section 4, the convergence analysis of the method is derived. To illustrate validity, accuracy, efficiency and applicability of the method some numerical examples are given and compared with the other methods [22,36,34,35,23] in Section 5. Finally, the conclusion is given in Section 6.

2. Shifted orthonormal Bernstein polynomials

The explicit representation of the orthonormal Bernstein polynomials of mth degree are defined on the interval $[0,1]$ as [33]

$$\psi_{j,m}(t)=\sqrt{2(m-j)+1}(1-t)^{m-j}\sum_{k=0}^{j}(-1)^k\binom{2m+1-k}{j-k}\binom{j}{k}t^{j-k},\quad j=0,\ldots,m.\tag{3}$$

In addition, (3) can be written in a simpler form in terms of original non-orthonormal Bernstein basis functions as [33]

$$\psi_{j,m}(t)=\sqrt{2(m-j)+1}\sum_{k=0}^{j}(-1)^k\frac{\binom{2m+1-k}{j-k}\binom{k}{j}}{\binom{m-k}{j-k}}B_{j-k,m-k}(t),\quad j=0,\ldots,m.$$

These polynomials satisfy the following orthogonality relation

$$\int_0^1\psi_{i,m}(t)\psi_{j,m}(t)dt=\delta_{i,j},\quad i,j=0,\ldots,m.$$

where $\delta_{i,j}$ is the Kronecker delta function.

To use of orthonormal Bernstein polynomials in the larger interval $[0,L]$, it is necessary to shift the defining domain $[0,1]$. The shifted orthonormal Bernstein polynomials on $[0,L]$ can easily be obtained by using the transformation $t=\frac{x}{L}$ in (3), that is

$$\varphi_{i,m}(x)=\frac{1}{\sqrt{L}}\psi_{i,m}\left(\frac{x}{L}\right).\tag{4}$$

By definition

$$\Phi(x)=[\varphi_{0,m}(x),\varphi_{1,m}(x),\ldots,\varphi_{m,m}(x)]^T,$$
$$\Psi(x)=[\psi_{0,m}(x),\psi_{1,m}(x),\ldots,\psi_{m,m}(x)]^T,$$

it is easily seen from (4) that

$$\Phi(x)=\frac{1}{\sqrt{L}}\Psi\left(\frac{x}{L}\right).\tag{5}$$