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Statistical inference for competing risks model in step-stress partially accelerated life tests with progressively Type-I hybrid censored Weibull life data

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ABSTRACT

In this paper, we discuss the maximum likelihood and Bayesian estimation under the progressively Type-I hybrid censored Weibull life data. The Weibull unknown parameters and acceleration factor in the step-stress partially accelerated life tests with competing risks are estimated based on the tampered failure rate model. The asymptotic confidence intervals and highest posterior density credible intervals are given by using the asymptotic normality theory and Gibbs sampling method. In particular, the acceptance–rejection algorithm is used to sample from the truncated density function, and the adaptive rejection algorithm is employed to sample from the log-concave density family. Finally, a simulation study is carried out to present simulated results and compare maximum likelihood estimates with Bayesian estimates. It is concluded that Bayesian estimation performs better.

1. Introduction

In life tests, it usually takes a long time to get the lifetime of products with high reliability under normal use conditions. Accelerated life test (ALT) can be used to obtain the failure information of a product under accelerated stress conditions in a short time. Generally, the acceleration factor is specified as a known function to explain the relationship between the reliability index and the accelerated stress in ALT. However, in some situations, the accelerated function is unknown and the ALT is not available. In this case, the products can be tested first under normal conditions until the prefixed time and then the surviving products are changed to put in accelerated stress conditions. This is partially accelerated life test (PALT).

There are two types of PALT, step-stress PALT (SSPALT) and constant-stress PALT (CSPALT). In CSPALT, the grouped test units are separately put in normal conditions and accelerated stress conditions. For SSPALT, the surviving items in the experiment are shifted from normal conditions to higher stress conditions after a fixed time or a fixed failure number. Although PALT can be conducted in the experiment to shorten test time, it still costs much time for the experimenter to wait for complete failure. It is necessary to take censoring schemes into account.

SSPALT has been addressed under different censoring schemes. Several authors have given statistical inferences and optimal design based on Type-I and Type-II censoring schemes, for example, see [1–8]. Ismail [9] also gave estimation of the Weibull parameters and acceleration factor under hybrid censoring. Based on the SSPALT with progressive censoring, Ismail [10] made inference in the generalized exponential distribution with progressive Type-II censoring. Sun

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et al. [11] elaborated inference for Burr-XII with progressive Type-II censoring with random removals. Abdel-Hamid and Al-Hussaini [12] presented inference and optimal design for the generalized Pareto distribution under progressive Type-I censoring.

Up to now, progressively Type-I hybrid censoring scheme (Type-I PHCS) proposed by Childs et al. [13] is becoming popular and widely used in life tests and ALTs for its flexible termination time. Similarly, it also can be applied in SSPALT. Recently, Ismail [14] has made likelihood inference from Weibull distribution with Type-I PHCS in SSPALT. Further, to explain the effect of changing the stress at the prefixed time in SSPALT, Bhattacharyya and Soejoeti [15] developed the tampered failure rate (TFR) model. This model has been used in ALTs, see [16,17]. Besides, Abdel-Hamid and Al-Hussaini [12] discussed statistical inference for the generalized Pareto distribution based on TFR model in SSPALT.

In reality, judging the effects of failure causes in SSPALT is equally important in addition to shortening testing time. The experimenter has to distinguish failure cause in order to obtain exactly failure information. In ALT, Wu et al. [18] discussed competing risks model under Type-I PHCS. In this paper, we aim to study statistical inference of step-stress partially accelerated life tests under Type-I PHCS in the presence of competing risks by using tampered failure rate model.

The remainder of this paper is arranged as follows. In Section 2, the model assumptions and description are elaborated and the likelihood function is presented. Maximum likelihood estimation (MLE) and asymptotic confidence intervals are derived in Section 3. Bayesian estimation and highest posterior density (HPD) credible intervals are obtained in Section 4. The simulation study and conclusion are given respectively in Sections 5 and 6.

2. Model description and likelihood function

This model is constructed to estimate parameters of independent Weibull failure causes and acceleration factor in SSPALT with progressively Type-I hybrid censoring data and the likelihood function of this model is expressed in this section.

2.1. Basic assumption and model description

Basic assumptions

(1) Two stress levels S_0 and S_1 are used in SSPALT, and $S_0 < S_1$.

(2) Under the stress level S_0, failure cause X_j, for $j = 1, 2, \ldots, p$, is considered with the hazard rate function (HRF)

$$h_j(x) = (\alpha_j/\eta_j)(x/\eta_j)^{\alpha_j - 1}, \quad x > 0;\ \alpha_j > 0,\ \eta_j > 0.$$

(3) The HRF for X_j under S_1 is proportional to $h_j(x)$ after the changing time τ, which is known as the TFR model developed by Bhattacharyya and Soejoeti [15].

(4) The total HRF of X_j in SSPALT is expressed as

$$h_j^*(x) = \begin{cases} h_{1j}(x) = h_j(x), & 0 < x \le \tau, \\ h_{2j}(x) = \lambda_j h_j(x), & x > \tau, \end{cases} \tag{1}$$

where $\lambda_j \ge 1$ is the acceleration factor and $\lambda_1 = \lambda_2 = \cdots = \lambda_p = \lambda \ge 1$. The corresponding reliability function $S_j^*(x)$, $j = 1, 2 \ldots p$, is given as

$$S_j^*(x) = \begin{cases} S_{1j}(x) = \exp\{-(x/\eta_j)^{\alpha_j}\}, & 0 < x \le \tau, \\ S_{2j}(x) = \exp\{-(\tau/\eta_j)^{\alpha_j} - \lambda_j[(x/\eta_j)^{\alpha_j} - (\tau/\eta_j)^{\alpha_j}]\}, & x > \tau. \end{cases} \tag{2}$$

(5) The latent failure time in the presence of independent failure causes is recorded as joint random variable (T, δ), where $T = \min(X_1, X_2, \ldots, X_p)$, $\delta = (c_1, c_2, \ldots, c_p)$ and for $j = 1, 2, \ldots, p$

$$c_j = \begin{cases} 1, & \text{if } T_i = X_j \\ 0, & \text{if } T_i \ne X_j. \end{cases}$$

Life test procedures under Type-I PHCS

(1) There are n items firstly put into the condition with normal stress S_0 in the SSPALT, then at the changing time τ, the surviving items are tested under acceleration stress level S_1.

(2) In SSPALT, the failure number m and censored time $T_0 > \tau$ are prefixed with the fixed removal number $(R_1, R_2, \ldots, R_m, R_\tau)$ satisfying $n = \sum_{i=1}^{m} R_i + R_\tau + m$. At the ith failure time $(t_{i:m:n}, \delta_i)$, $R_i \ge 0$ items would be withdrawn from the surviving ones and $R_\tau \ge 0$ items would be removed at the changing time τ. Until the time $\min(t_{m:m:n}, T_0)$, the left items are censored and the test is terminated. Therefore, we have the observed data in the SSPALT under Type-I PHCS

$$S_0 : (t_{1:m:n}, \delta_1, R_1), (t_{2:m:n}, \delta_2, R_2), \ldots, (t_{r_1:m:n}, \delta_{r_1}, R_{r_1}), (\tau, R_\tau),$$
$$S_1 : (t_{r_1+1:m:n}, \delta_{r_1+1}, R_{r_1+1}), (t_{r_1+2:m:n}, \delta_{r_1+2}, R_{r_1+2}), \ldots, (t_{r_1+r_2:m:n}, \delta_{r_1+r_2}, R_{r_1+r_2}), (T_0, R_c),$$

where r_1 and r_2 are the random failure numbers respectively under the normal stress level S_0 and acceleration stress level S_1 and $t_{1:m:n} < t_{2:m:n} < \cdots < t_{r_1:m:n} \le \tau < t_{r_1+1:m:n} < t_{r_1+2:m:n} < \cdots < t_{r_1+r_2:m:n} < T_0$, $\delta_i = (c_{i1}, c_{i2}, \ldots, c_{ip})$, $i = 1, 2, \ldots, r_1 + r_2$.