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A two-step model reduction approach for mechanical systems with moving loads

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ABSTRACT

We consider model order reduction of mechanical systems with moving loads. Such systems have a time-varying input matrix that makes the direct application of standard model reduction methods difficult. In this paper, we present a two-step model reduction approach for systems with moving loads which is based on a low-rank approximation of the input matrix and applying Krylov subspace methods to the resulting linear time-invariant system with a modified input. Numerical results demonstrate the properties of the proposed model reduction method.

1. Introduction

In structural dynamics, the moving load problem has received a lot of attention because of its importance in many practical applications. Mechanical systems with moving loads arise, for example, in modelling of bridges with moving vehicles [1,2], cableways [1], cranes [3] or working gears [4]. One of the most popular methods for simulation of the dynamic behaviour of such systems is a finite element method (FEM) based on a variational formulation of the structural mechanics problem, e.g., [5]. In the engineering literature, the principle of virtual work is used to derive the FEM approximations, see [6]. The FEM discretization of a system subjected to moving loads yields a linear time-varying (LTV) second-order system

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= B(t)u(t), \\ y(t) &= C(t)q(t), \end{aligned} \tag{1}$$

where $M, D, K \in \mathbb{R}^{N\times N}$ are the mass, damping and stiffness matrices, respectively, $q(t) \in \mathbb{R}^N$ is an unknown vector of generalized coordinates, $u(t) \in \mathbb{R}^m$ is the input, and $y(t) \in \mathbb{R}^p$ is an output describing a response in a domain of interest. Furthermore, $B(t) \in \mathbb{R}^{N\times m}$ and $C(t) \in \mathbb{R}^{p\times N}$ are the time-dependent input and output matrices describing, respectively, force and observation positions at time $t \in [0, T]$.

As an example, we consider a simply supported beam excited by a moving force. The vibration of the beam is modelled by an Euler–Bernoulli equation

$$\mu A \frac{\partial^2}{\partial t^2} w(x,t) + 2\mu A \omega_{\mathrm{d}} \frac{\partial}{\partial t} w(x,t) + EJ \frac{\partial^4}{\partial x^4} w(x,t) = f(x,t) \tag{2a}$$

with the initial and boundary conditions

$$w(x,0) = 0, \qquad \frac{\partial}{\partial t} w(x,0) = 0, \tag{2b}$$

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$$w(0,t)=0, \qquad \frac{\partial^2}{\partial x^2}w(0,t)=0, \qquad w(l,t)=0, \qquad \frac{\partial^2}{\partial x^2}w(l,t)=0, \tag{2c}$$

where $w(x,t)$ is a vertical displacement of the beam, μ is the mass density, A is the cross-sectional area, E is the Young modulus, J is the moment of inertia of the cross-section, ω_d is the circular frequency of damping, and l is the length of the beam. The external force acting on the beam is described by a function

$$f(x,t)=\sum_{j=1}^{m}\varrho_j(x,t)u_j(t),$$

where $\varrho_j(x,t)$ and $u_j(t)$ are the normalized distribution and magnitude of the jth excitation, respectively. The time dependence in $\varrho_j(x,t)$ indicates a time-varying excitation or a moving load. Applying the FEM to (2), we get system (1) with the symmetric, positive definite matrices M, D and K. The entries of the input matrix $B(t)=[b_{ij}(t)]$ have the form

$$b_{ij}(t)=\int_0^l \varrho_j(x,t)\,\varphi_{N,i}(x)\,dx, \quad i=1,\ldots,N,\ j=1,\ldots,m, \tag{3}$$

where $\varphi_{N,i}(x)$ are the FEM basis functions and N is the number of degrees of freedom. If the beam is excited by the point forces, then

$$\varrho_j(x,t)=\delta(x-\xi_j(t)),$$

where $\delta(x)$ is the Dirac delta function and $\xi_j(t)$ is an instantaneous position of the jth force at time t. In this case, the entries of $B(t)$ are defined especially simply as

$$b_{ij}(t)=\varphi_{N,i}(\xi_j(t)).$$

Unfortunately, simulation of complex mechanical structures requires frequently an overwhelming computational effort due to a large number of degrees of freedom. In order to reduce the computational complexity when solving the large-scale system (1) numerically, we may use model order reduction. It consists of approximating (1) by a reduced model

$$\begin{aligned} \widetilde{M}\ddot{\widetilde{q}}(t)+\widetilde{D}\dot{\widetilde{q}}(t)+\widetilde{K}\,\widetilde{q}(t) &= \widetilde{B}(t)\,u(t),\\ \widetilde{y}(t) &= \widetilde{C}(t)\,\widetilde{q}(t), \end{aligned} \tag{4}$$

where $\widetilde{M},\widetilde{D},\widetilde{K}\in\mathbb{R}^{r\times r}$, $\widetilde{B}(t)\in\mathbb{R}^{r\times m}$, $\widetilde{C}(t)\in\mathbb{R}^{p\times r}$ and $r\ll N$. Such a model can be computed by projection

$$\widetilde{M}=W^TMV, \qquad \widetilde{D}=W^TDV, \qquad \widetilde{K}=W^TKV, \qquad \widetilde{B}(t)=W^TB(t), \qquad \widetilde{C}(t)=C(t)V,$$

where the projection matrices $V,W\in\mathbb{R}^{N\times r}$ are chosen such that the approximation error $\widetilde{y}-y$ is small in an appropriate norm. For linear time-invariant (LTI) second-order systems, the projection matrices can be determined using balanced truncation [7–9] or (rational) Krylov subspace techniques [10–12]. The balanced truncation method has been extended to LTV systems in [13,14]. It relies on solving two Lyapunov differential equations and is computationally expensive and memory demanding. Another model reduction approach for LTV systems has been presented in [15,16]. It is based on converting the LTV system to a LTI system by time discretization and applying the recycled Krylov subspace technique. Note that these both model reduction approaches are applicable to general LTV systems, where all system matrices are time-varying. In system (1), however, the state matrices M, D and K are time-independent, and only the input and output matrices depend on time. Such a system can be reformulated as the LTI system

$$\begin{aligned} M\ddot{q}(t)+D\dot{q}(t)+Kq(t) &= u_{\mathrm{new}}(t),\\ y_{\mathrm{new}}(t) &= q(t) \end{aligned} \tag{5}$$

with the input $u_{\mathrm{new}}(t)=B(t)u(t)\in\mathbb{R}^N$ and the output $y_{\mathrm{new}}(t)\in\mathbb{R}^N$ of large dimension. Model reduction of systems with many inputs and outputs has been considered in [17–19]. The methods proposed there are based on a low-rank approximation of the input and output matrices using a singular value decomposition combined with standard model reduction techniques. Unfortunately, these methods cannot be applied to system (5), since the input and output matrices in (5) are the identity matrices. In [10,20], an $\mathscr{H}_2$-optimal model reduction method has been developed which is based on an iterative rational Krylov algorithm (IRKA). This method can also be applied to systems with many inputs and outputs. It should, however, be noticed that for such systems, IRKA may exhibit a slow convergence or even the iteration may stagnate.

Model reduction of systems with moving loads has been considered in [21–23], where the variability of the external force was described either by switched systems or by parameter-dependent systems. However, these modelling approaches neglect to some extent the time variation of the load location. In this paper, we use a time-varying input matrix to model moving loads and present a two-step model reduction method for the resulting LTV system. Our approach is based on a transformation of the LTV system (1) into a LTI system by approximating the input matrix and application of structure-preserving Krylov subspace model reduction methods. Note that this model reduction approach is not restricted to mechanical systems with moving loads and can also be applied to other equations with moving sources or boundary conditions. An example includes heat conduction problems with moving heat sources arising in numerous engineering applications such as laser hardening, welding, cutting and grinding [24,25].