Journal of Computational and Applied Mathematics 297 (2016) 98–116

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A new finite difference method for pricing and hedging fixed income derivatives: Comparative analysis and the case of an Asian option[☆]

Allan Jonathan da Silva [a], Jack Baczynski [a,*], José V.M. Vicente [b,c]

[a] National Laboratory for Scientific Computing, LNCC, 25651-075, Petrópolis, RJ, Brazil
[b] Central Bank of Brazil, 20071-000, Rio de Janeiro, RJ, Brazil
[c] Ibmec Business School, 20071-000, Rio de Janeiro, RJ, Brazil

ARTICLE INFO

Article history:
Received 17 August 2015
Received in revised form 21 October 2015

Keywords:
Interest rates derivatives
Option pricing and hedging
PDE
Finite difference methods
Computational finance

ABSTRACT

We propose a second order accurate numerical finite difference method to replace the classical schemes used to solve PDEs in financial engineering. We name it Modified Fully Implicit method. The motivation for doing so stems from the accuracy loss while trying to stabilize the solution via the up-wind scheme in the convective term as well as the fact that spurious oscillations solutions occur when volatilities are low (this is actually the range that is commonly observed in interest rate markets). Unlike the classical schemes, our method covers the whole spectrum of volatilities in the interest rate dynamics.

We obtain analytical and numerical results for pricing and hedging a zero-coupon bond and an Asian interest rate option. In the case of the Asian option, we compare the realistic discrete compounding interest rate scheme (associated with the Modified Fully Implicit method) with the continuous compounding scheme (often exploited in the literature), obtaining relative discrepancies between prices exceeding 50%. This indicates that the former scheme is more appropriate then the latter to price more complicate derivatives than straight bonds.

1. Introduction

Before the 1980s, fixed income markets were composed primarily by vanilla bonds and simple structured financial instruments. Thus, their valuations were easy and direct, done frequently via closed-form mathematical formulas (e.g. [1]). Thenceforth, markets have become sophisticated as more complex products aiming to reduce or share risks appear, complicating the pricing and hedging engines. The fast growth of financial market instruments over recent decades has spawned many challenging mathematical problems to be solved, from the underlying stochastic modeling to solutions through computational methods.

Fixed income derivatives are contracts which have payoff, contingent on the evolution of interest rates. They are traded in the equity, commodity, currency and credit markets along with hybrid derivatives engineered over the counter [2]. The valuation of interest rate derivative contracts is a very important subject in modern financial theory and practice. The

☆ We thank useful comments from seminar participants at LNCC (2015), the 14th Brazilian Meeting of Finance (2014) and the 35th National Conference on Computational and Applied Mathematics (2014).

* Corresponding author.
E-mail addresses: allanjs@lncc.br (A.J. da Silva), jack@lncc.br (J. Baczynski), jose.valentim@bcb.gov.br (J.V.M. Vicente).

http://dx.doi.org/10.1016/j.cam.2015.10.025
0377-0427/

financial health of banks, governments and industrial companies are very sensitive to changes in the term structure of the interest rates. It has become mandatory nowadays to quantify and control the risk exposure to prices of interest rate associated contracts.

A large amount of academic literature has been dedicated to the pricing and hedging of such instruments. Vasicek [3] has introduced a Gaussian stochastic process to model the spot rate dynamics. He also developed a simple closed-form solution to compute the prices of zero-coupon bonds. Jamshidian [4] has extended the results to options on bonds, which is automatically adapted to price interest rate caps and floors and Hübner [5] used Jamshidian's approach to express the swaption prices. Additionally, closed-form expressions have been developed to price such products based on other stochastic processes (see e.g. [6–8]).

However, it is a hard task to extend the results and find analytical solutions to more complex structures, even in the Gaussian model. The callable bond is an example. It is a financial instrument commonly issued by banks and non-financial companies. Hence, such contracts must be priced by numerical techniques. Several computational approaches, such as Fourier methods [9], Monte Carlo simulation [10] and tree methods [11] can be used to price complex derivatives, but due to its efficiency in computing accurate pricing and hedging values and its flexibility in the modeling process, partial differential equations have become a very popular choice. Some recent developments in the field of financial engineering designed for specific purposes can be found (i) in [12], for pricing discrete double barrier option via PDE transforms, (ii) in [13], for pricing discrete monitored barrier options in the Black–Scholes scenario, where the authors mix the Laplace Transform and the finite difference method, (iii) in [14], where a meshfree method is used to calculate the prices and the greeks of European, Asian and Barrier options, and (vi) in [15], where quadrature methods are applied to price discretely monitored Barrier options.

To improve financial engineering, we propose a new numerical finite difference method to replace the classical schemes used to solve PDEs (see e.g. [16–18]). The motivation for doing so stems from the fact that spurious oscillations solutions occur when volatilities are low (i.e., when the Peclet number is high) and serious collateral matters appear in attempts to correct the problem. Actually, low volatilities are the range observed in interest rate markets, and unlike the classical schemes, our method covers the whole spectrum of volatilities in the interest rate dynamics.

Our method is devised as a version of the Fully Implicit method (see, e.g., [16,18]), and extended to provide hedges along with prices. One of the modifications we introduced is inspired in a technique that appears in [19]. That method adapts to the Black–Scholes dynamics, while ours fit the interest rate derivatives with Vasicek, CIR [7] and other types of short-rate models. Our numerical scheme is first order accurate in time, second order accurate in space and consistent. Moreover, it possesses the quality of being unconditionally stable. We name it Modified Fully Implicit (Interest Rate) Method.

We show the good performance of the method, pricing a zero-coupon bond and another type of interest rate derivative security named IDI (Interbank Deposit Rate Index) option, both in the Vasicek dynamic. Namely, we perform a convergence analysis by considering both continuously compounded and daily compounded rate of interest to model the money market account and the updating of the IDI path.[1]

The ID index updating is built up discretely based on the overnight DI rate, which is an annualized rate over one day period. It is calculated and published daily, and represents the average rate of inter-bank overnight transactions [20]. Based on a martingale approach, closed form solutions to price an IDI contract are available in the literature, assuming for mathematical tractability reasons that the updating of the IDI is continuous in time. In this scenario, a one-factor model was developed in [21] to price the IDI option via the short rate dynamics as given in [3]. A multi-factor Gaussian model was developed in [22] to price the IDI option and bond prices. Also, [23] proposed to incorporate the potential changes in the targeting rates via pure jump process.

Carrying on the evaluation of our finite difference scheme, we demonstrate its advantages considering the following approaches on a pricing problem of an IDI call option with the Vasicek dynamic.

- We obtain the estimates of the prices (and hedges) according to the Modified Fully Implicit method, and consider updating the IDI path discretely. This updating rule allow us to track realistically the evolution of the index and to achieve the exact pay-off representation.
- We obtain the prices via the closed form expressions given in [21], assuming a continuously compounded interest rate, which is actually an idealization for mathematical tractability.

So, our approach corresponds to obtaining approximate prices for the exact problem (with respect to the payoff) while that of [21] corresponds to obtaining an exact price for the approximate problem. The results of this comparative analysis corroborate the conjecture of Tankov and Cont [24], which asserts that, typically, the former scenario yields better results than the latter. Indeed, via numerical simulations, we observe meaningful relative discrepancies in the prices for some prescribed examples whose parameters are good representatives of the market. So, using one or other method makes a difference. Now, neither price represents a benchmark. The benchmark should correspond to a framework that models the

[1] IDI is the shorthand of Interbank Deposit Rate Index. The IDI option is a financial option of Asian type and, as such, the payoff depends on the path followed by the short term interest rate. It presents cheaper prices than the standard options and it is less sensitive to extreme market conditions that may prevail close to the expiration day—due to random crashes or outright manipulation. So, it is commonly used by corporations to manage interest rate risk. Actually, it is a standardized derivative product traded at the Securities and Futures Exchange in the Brazilian fixed income market.