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On the local convergence and the dynamics of Chebyshev–Halley methods with six and eight order of convergence

Á. Alberto Magreñán [a,*], Ioannis K. Argyros [b]

[a] Universidad Internacional de La Rioja (UNIR), Gran Vía Rey Juan Carlos I, 41, 26002 Logroño, La Rioja, Spain
[b] Cameron University, Department of Mathematics Sciences Lawton, OK 73505, USA

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ABSTRACT

We study the local convergence of Chebyshev–Halley methods with six and eight order of convergence to approximate a locally unique solution of a nonlinear equation. In Sharma (2015) (see Theorem 1, p. 121) the convergence of the method was shown under hypotheses reaching up to the third derivative. The convergence in this study is shown under hypotheses on the first derivative. Hence, the applicability of the method is expanded. The dynamics of these methods are also studied. Finally, numerical examples examining dynamical planes are also provided in this study to solve equations in cases where earlier studies cannot apply.

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1. Introduction

In this study we are concerned with the problem of approximating a locally unique solution x^* of the equation

$$F(x) = 0, \tag{1.1}$$

where F is a differentiable function defined on a convex subset D of S with values in S, where S is $\mathbb{R}$ or $\mathbb{C}$.

Numerous problems from Applied Sciences including engineering can be solved by means of finding the solutions of equations in a form like (1.1) using Mathematical Modeling [1–4]. Except in special cases, the solutions of these equations can be found in closed form. This is the main reason why the most commonly used solution methods are usually iterative. The convergence analysis of iterative methods is usually divided into two categories: semilocal and local convergence analysis. The semilocal convergence matter is, based on the information around an initial point, to give criteria ensuring the convergence of iteration procedures. A very important problem in the study of iterative procedures is the convergence domain. In general the radius of convergence is small. Therefore, it is important to enlarge the radius.

The dynamical properties related to an iterative method applied to polynomials give important information about its stability and reliability. In recent studies, authors such as Cordero et al. [5–9], Amat et al. [10,11,2], Gutiérrez et al. [12],

* Corresponding author.
E-mail addresses: alberto.magrenan@unir.net (Á.A. Magreñán), iargyros@cameron.edu (I.K. Argyros).

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Chun et al. [6], Magreñán [13,14], and many others [15–21,12,22–29,4,30,31] have found interesting dynamical planes, including periodical behavior and others anomalies. One of our main interests in this paper is the study of the parameter spaces associated to a family of iterative methods, which allow us to distinguish between the good and bad methods in terms of its numerical properties. These are with the study of the dynamical behavior our objectives in this paper.

Recently, J. Sharma in [4] studied the local convergence of the method defined for each $n = 0, 1, 2, \ldots$ by

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ z_n &= x_n - (1 + (F(x_n) - 2\alpha F(y_n))^{-1}F(y_n))F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= x_n - A_n^{-1}F(z_n), \end{aligned} \tag{1.2}$$

where x_0 is an initial point, $\alpha \in S$ a given parameter and

$$A_n = [x_n, y_n; F] + [z_n, y_n, x_n; F](z_n - y_n) + [z_n, y_n, x_n, x_n; F](z_n - y_n)(z_n - x_n)$$

and $[x_n, y_n; F]$, $[z_n, y_n, x_n; F]$, $[z_n, y_n, x_n, x_n; F]$ are divided differences of order one, two, three [1–3,30] respectively defined by

$$\begin{aligned} [z, y; F] &= \frac{F(z) - F(y)}{z - y}, \\ [z, y, x; F] &= \frac{[z, x; F] - [y, x; F]}{z - y}, \\ [z, y, x, x; F] &= \frac{[z, x, x; F] - [y, x, x; F]}{z - y} \end{aligned}$$

where

$$[z, x, x; F] = \frac{[z, x; F] - F'(x)}{z - x}.$$

The order of convergence was shown to be at least six and if $\alpha = 1$, then the order of convergence is eight.

This method includes the modifications of Chebyshev's method ($\alpha = 0$), Halley's method ($\alpha = 1/2$) and super-Halley method ($\alpha = 1$). Method (1.2) is a useful alternative to the third order Chebyshev–Halley-methods [19–21,12,22–24] defined for each $n = 0, 1, 2, \ldots$ by

$$x_{n+1} = x_n - \left(1 + \frac{1}{2}(1 - \alpha K_F(x_n))^{-1}K_F(x_n)\right)F'(x_n)^{-1}F(x_n), \tag{1.3}$$

where

$$K_F(x_n) = F'(x_n)^{-1}F''(x_n)F'(x_n)^{-1}F(x_n),$$

since the computation of $F''(x_n)$ is being avoided.

Method (1.2) can also be used instead of a method by D. Li, P. Liu and J. Kou [27] defined for each $n = 0, 1, 2, \ldots$ by

$$\begin{aligned} y_n &= x_n - F'(x_n)^{-1}F(x_n) \\ z_n &= x_n - (1 + (F(x_n) - 2\alpha F(y_n))^{-1}F(y_n))F'(x_n)^{-1}F(x_n) \\ x_{n+1} &= z_n - B_n^{-1}F(z_n), \end{aligned} \tag{1.4}$$

where

$$B_n = F'(x_n) + \bar{F}''(x_n)(z_n - x_n)$$

and

$$\bar{F}''(x_n) = 2F(y_n)F'(x_n)^2F(x_n)^{-2}.$$

However, the convergence of the preceding methods has been shown using Taylor expansions under hypotheses on at least the third derivative (see Theorem 1 in [4]) which limits the applicability of these methods, although only the first derivative appears in method (1.2). Notice, that for the convergence order of method (1.2) the existence of the ninth derivative is required. As a motivational example, define function F on $\mathbb{X} = \mathbb{Y} = \mathbb{R}$, $D = \bar{U}(0, 1)$ by

$$F(x) = \begin{cases} c_1x^3\ln x^2 + c_2x^5 + c_3x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

where $c_1 \neq 0$, c_2 and c_3 are real parameters. Then, we have that

$$\begin{aligned} F'(x) &= 3c_1x^2\ln x^2 + 5c_2x^4 + 4c_3x^3 + 2c_1x^2, \\ F''(x) &= 6c_1x\ln x^2 + 20c_2x^3 + 12c_3x^2 + 10c_1x \end{aligned}$$

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