



Iterative algorithm for the first passage time distribution in a jump–diffusion model with regime-switching, and its applications



Jerim Kim^a, Bara Kim^{b,*}, In-Suk Wee^b

^a Department of Business Administration, Yong In University, 134 Yongindaeak-ro, Cheoin-gu, Yongin-si, Gyeonggi-do, 449-714, Republic of Korea

^b Department of Mathematics, Korea University, 145, Anam-ro, Seongbuk-gu, Seoul, 136-701, Republic of Korea

ARTICLE INFO

Article history:

Received 2 July 2014

Received in revised form 26 June 2015

MSC:

60J22

60J25

65C40

Keywords:

First passage time

Laplace transform

Iterative algorithm

Jump–diffusion

Regime-switching

Defaultable bond pricing

ABSTRACT

For a regime-switching model with a finite number of regimes and double phase-type jumps, Jiang and Pistorius (2008) derived matrix equations with real parameters for the Wiener–Hopf factorization. The Laplace transform of the first passage time distribution is expressed in terms of the solution of the matrix equations. In this paper we provide an iterative algorithm for solving the matrix equations of Jiang and Pistorius (2008) with complex parameters. This makes it possible to obtain numeric values of the Laplace transform with complex parameters for the first passage time distribution. The Laplace transform with complex parameters can be inverted by numerical inversion algorithms such as the Euler method. As an application, we compute the prices of defaultable bonds under a structural model with regime switching and double phase-type jumps.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

First passage time distributions for Lévy processes have been investigated for several decades by many authors. One of the useful tools for studying the first passage time distributions is the celebrated Wiener–Hopf factorization. See, for example, [1] or [2] for the Wiener–Hopf factorization of Lévy processes. By using the Wiener–Hopf factorization, Rogers [3] derived an explicit expression for the Laplace transform of the first passage time distribution of a Lévy process with one-sided jumps. He applied a variant of the method developed by Abate and Whitt [4] and others to numerically invert the Laplace transform. The Laplace transform of the exit time from an interval was derived by Avram, Kyprianou and Pistorius [5] for a Lévy process with one-sided jumps. For general Lévy processes, Doney and Kyprianou [6] expressed the quintuple law describing the time of the first passage, the time of the last maximum before the first passage, the overshoot, the undershoot and the undershoot of the last maximum, in terms of the Lévy measure and the distributions of the ascending and descending ladder processes.

Lévy processes of jump–diffusion type are special Lévy processes that have finite jump activity. They have been widely used in financial modeling since Merton [7] employed a jump–diffusion process with normal jumps as a model of the log price of an asset. Zhou [8] used a Monte Carlo approach for the first passage time distribution of the jump–diffusion process

* Corresponding author.

E-mail addresses: jerimkim@yongin.ac.kr (J. Kim), bara@korea.ac.kr (B. Kim), iswee@korea.ac.kr (I.-S. Wee).

with normal jumps to study the term structure of credit spreads with jump risk. For a jump–diffusion process with double exponential jumps, Kou and Wang [9] derived an explicit expression for the Laplace transform of the first passage time distribution, and Kou [10] and Kou and Wang [11] derived analytic solutions for prices of European vanilla options and path dependent options such as lookback, barrier and perpetual American options. Cai and Kou [12] extended the Kou model to a jump–diffusion process with double hyper-exponential jumps. They derived the Laplace transform of the exit time from an interval and then obtained the Laplace transform of the price of a double-barrier option. Yin, Shen and Wen [13] investigated downward first passage time distributions for jump–diffusion processes with hyper-exponential downward jumps and general upward jumps. For the jump–diffusion process with double phase-type jumps, Asmussen, Avram and Pistorius [14] derived explicit expressions for the Wiener–Hopf factorization and the Laplace transform of the first passage time distribution. Using the Laplace transform of the first passage time distribution, they derived explicit expressions for the prices of the American perpetual put and Russian options.

Analytic solutions for the first passage time distributions are known for some regime-switching models. When there are two regimes, Guo and Zhang [15] obtained a closed form solutions for the first passage time distributions and the price of the perpetual American put option, and Hieber [16] extended their results to three regimes. Kijima and Siu [17] expressed the first passage time distribution as a solution of a system of linear equations under the double exponential jump–diffusion model with two regimes. For the regime-switching model with two regimes and double phase-type jumps, Asmussen, Avram and Pistorius [14] obtained analytic solution for the first passage time distribution. Carr and Crosby [18] derived semi-closed solution of the first passage time distribution for a particular regime-switching Lévy model. The Wiener–Hopf factorization was studied by Jiang and Pistorius [19] for the regime-switching model with a finite number of regimes and double phase-type jumps, and by Breuer [20] for the regime-switching model with a finite number of regimes, positive jumps of phase-type and general negative jumps.

The analysis of the first passage time has many applications. For applications to the analysis of barrier options, refer to Cont and Voltchkova [21] and Jeannin and Pistorius [22]. For applications to the analysis of risk models, refer to Gerber and Landry [23], Gerber and Shiu [24] and Kluppelberg, Kyprianou and Maller [25]. Applications to the analysis of credit risk models can be found in [26–28]. Benedetto, Sacerdote and Zucca [29] applied the analysis of the first passage time to a model of neuroscience. Many other applications can be found in the literature.

In this paper we consider a regime-switching model with a finite number of regimes and double phase-type jumps. This is the same model as considered by Jiang and Pistorius [19]. They investigated the Wiener–Hopf factorization which is closely linked to the first passage time distribution. They derived matrix equations for the Wiener–Hopf factorization. They solved the matrix equations in an example only when there are two regimes, jumps in one regime are (one-sided) exponential and no jumps occur in the other regime. For practical applications of their results with general regime-switching model of double phase-type jumps, it is required to obtain numeric values of the solution for the matrix equations. In this work, we provide an iterative algorithm for solving the matrix equations. By using the algorithm we can obtain numeric values of the Laplace transform with complex parameters for the first passage time distribution.

The first passage time distributions can be used for pricing of several path dependent options such as barrier, lookback, perpetual American put and Russian options. Other applications of the first passage time distributions can also be found in pricing of defaultable bonds based on structural models. As an application of the computation of the Laplace transform of the first passage time distribution in the jump–diffusion model with regime-switching, we derive a formula for the Laplace transform of the price of defaultable bonds. The Laplace transform with complex parameters can be numerically inverted with the help of commonly used inversion algorithm. Numerical inversion method developed by Abate and Whitt [4] is used to calculate the price of the double-barrier option.

We remark that Breuer [20] also studied an iterative algorithm for calculation of the Laplace transform of the first passage time in a regime-switching model with jumps. He assumed that the parameters for the Laplace transform of the first passage time are real, even though we believe that, after a slight modification, his iterative algorithm works well for complex parameters with nonnegative real parts. It is noticed that Laplace transforms with complex parameters are needed to be calculated to apply the well-performed numerical inversion such as the Euler method. The present paper provides an algorithm for calculation of Laplace transforms with complex parameters. In a numerical example, Laplace transforms with complex parameters obtained by the algorithm are used for numerical inversion by the Euler method.

It is well known that the set of all double phase-type distributions is dense in the space of all distributions in the real line with Prokhorov topology. In this respect, the results of this paper on the first passage time distribution in the regime-switching model with double phase-type jumps can be used for an approximation of the first passage time distribution in the regime-switching model with general jump size distributions. We refer to Egami and Yamazaki [30] for an example of approximation methods using phase-type distributions.

The remainder of this paper is organized as follows. In Section 2, we describe our jump–diffusion model with regime-switching in detail. In Section 3, an iterative algorithm is provided for the Laplace transform of the first passage time distribution with complex parameters. Numerical examples are provided to show that our iterative algorithm works fast with complex parameters. Validity of the algorithm is theoretically proved in Section 4. Section 5 considers pricing of defaultable bonds in the jump–diffusion model with regime-switching. We apply our computation algorithm to obtain numeric values of the Laplace transform of the prices of the defaultable bonds. The prices of the defaultable bonds are obtained by the Euler inversion of the Laplace transform. Numerical examples are provided for pricing of the defaultable bonds.

Download English Version:

<https://daneshyari.com/en/article/4638114>

Download Persian Version:

<https://daneshyari.com/article/4638114>

[Daneshyari.com](https://daneshyari.com)