Journal of Computational and Applied Mathematics 294 (2016) 358–371

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Stability analysis of the marching-on-in-time boundary element method for electromagnetics

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ARTICLE INFO

Article history:
Received 27 June 2014

Keywords:
Time-dependent boundary integral equations
Boundary element method
Variational formulation
Collocation
Stability

ABSTRACT

The Time Domain Integral Equation method for electromagnetics is an appealing computational method for many applications in industry. However, its applicability has long been suffering from instabilities. A rigorous analysis of the variational formulation is imperative to the successful design of stable and robust numerical schemes. In this paper, an established functional framework and stability theorem will be extended to the differentiated version of the electric field integral equations, which can be discretized more efficient and is more often used in engineering literature. The extended stability theorem, combined with efficiency requirements, will give guidelines on the choice of test and basis functions of the space–time Petrov–Galerkin scheme. A discrete equivalence with the collocation method results in the recommendation to choose the quadratic spline basis function in the standard Marching-on-in-Time scheme. Computational experiments confirm that the quadratic spline basis functions have superior stability characteristics compared to the conventional quadratic Lagrange basis functions in time.

1. Introduction

Computational methods for transient electromagnetic scattering phenomena have widespread applications in industry, for instance in the design of aircraft or microelectronic circuits. The use of a boundary integral equation method is an appealing choice, because the number of degrees of freedom scale quadratically with the electrical size of the object and no artificial boundary to truncate the computational domain is required. The formulation in time domain allows for efficient computations of wideband signals and the incorporation of nonlinear models for material characteristics. Nevertheless, the applicability of time-domain boundary integral equation (TDIE) methods to computational electromagnetics has been limited due to inefficiency and instability. With the development of computational accelerators based on plane-wave and fast Fourier transform techniques, large-scale structures have been simulated [1,2]. However, obtaining robust and stable simulations is still a major challenge.

Whereas the Galerkin discretization in space has been the *de facto* standard, no consensus has been reached yet on the numerical discretization in time. Collocation [1], space–time Galerkin [3] and convolution quadrature [4] are the most popular choices. In this paper, collocation, also called Marching-on-in-Time (MoT), will be used because of its efficiency and accuracy. However, this numerical scheme has a long history of instability and has defied many stabilization techniques.

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http://dx.doi.org/10.1016/j.cam.2015.09.002
0377-0427/

Hence, stability of MoT schemes is an active topic of research [5–7] and this paper aims to provide a rigorous analysis of the stability of TDIE methods.

The instabilities encountered in TDIE methods can be subdivided into spectral and numerical instabilities. The spectral instabilities originate from the nontrivial null space of the continuous model equations, which consists of direct currents and resonances. These instabilities have successfully been eliminated with several techniques, for example Calderón preconditioning [8]. On the other hand, the numerical instabilities originate from the numerical discretization of the model equations and have been more persistent. These instabilities are typically noticed in simulations as exponentially increasing solutions that alternate on discrete time levels. Because the instability often plagues the solution only when the simulation time is long compared to the time scale of the excitation, it is also called late-time instability.

The model equations are given by retarded potential boundary integral equations and have a strong coupling of space and time. This prevents a straightforward use of classical stabilization techniques for ordinary and delayed differential equations. Remedies for numerical instability have to be specifically designed for TDIE methods. The most successful strategies are the following:

1. Suppression of high-frequency content: because the numerical instability is usually visible as a solution that alternates on discrete time levels, suppression of high-frequent content can alleviate the instability. This is the basis of filtering and averaging techniques [9], the use of large time step sizes [10] and the use of bandlimited temporal basis functions [11]. However, these techniques do not eliminate the instability entirely and impinge on efficiency and accuracy.
2. Improvement of numerical accuracy: as a boundary element method, the expressions of the elements of the discretization matrix contain surface integrals. Due to discontinuities in the integrand, standard quadrature techniques are inaccurate. The use of large CFL numbers [10], smooth temporal basis functions [12] and a separable approximation of convolutions [6] can improve the accuracy of evaluating the surface integrals. Furthermore, the use of quasi-exact integration techniques allows for very accurate evaluation of the elements of the discretization matrix and computational experiences confirm that this is necessary to obtain stability for MoT schemes [13,5,7].
3. Design of a functional framework: to obtain a stable numerical scheme, the solution of the variational formulation has to be bounded, which should be proven within a specific functional framework [14,15]. This is the most rigorous approach to eliminate numerical instability and will be adopted in this paper.

A thorough mathematical foundation for TDIE methods using space–time Galerkin discretization schemes has been given by Terrasse [16]. There, a functional framework for the time-domain Electric Field Integral Equation (EFIE) has been designed, based on the earlier work of Bamberger and Ha-Duong on scalar TDIE methods for acoustics [17]. The properties of the time-domain EFIE have been proven with a passage via the Laplace domain. This is also the approach commonly used in further research, see e.g. [18] for an overview. In this paper, the analysis of the TDIE method will be performed entirely in time-domain in order to stay close to the time-domain variational formulation that will be used to discretize the model equations. This makes the implications of the type of model equation on the choice of numerical scheme immediately visible.

In Terrasse [16], specific Sobolev spaces are designed such that uniqueness and boundedness of the solution of the variational formulation can be proven. This has been performed for the Electric Field Integral Equation (EFIE). The aim of this paper is to extend the stability theorem to the differentiated version of the EFIE. This version is broadly used in engineering literature and allows for more efficient discretization schemes [11,8,6]. However, the extended stability theorem cannot be applied directly to the efficient collocation scheme. This will be remedied by considering Petrov–Galerkin schemes that fit within the functional framework. These schemes will be shown to be discretely equivalent with collocation schemes with specific temporal basis functions. Since stability carries over to discretely equivalent schemes, an MoT scheme with quadratic spline basis functions that fits within the functional framework of the stability theorem will be used [19]. Computational experiments will confirm the stability.

2. Methodology

This paper focuses on the TDIE method, which is a boundary integral equation method in time domain for electromagnetic scattering analysis. The model equations, variational formulation, and the numerical discretization will be explained in Sections 2.1–2.3, respectively.

2.1. Model equations

Let us consider a bounded domain $\Omega^i \subset \mathbb{R}^3$ that represents the scatterer and $\overline{\Omega^i}$ its closure. The exterior domain $\Omega^e = \mathbb{R}^3 \setminus \overline{\Omega^i}$ represents free space. The unit outward normal vector $\hat{\mathbf{n}}$ on the interface $\Gamma = \partial\Omega^i = \partial\Omega^e$ points from Ω^i towards Ω^e. The scatterer is assumed to be a perfect electric conductor (PEC), for which the jump condition states that the tangential component of the electric field vanishes on the interface with free space. As initial condition, the incident wave field has not yet induced a current distribution on the scatterer at zero time. Then, the exterior scattering problem can be formulated as a boundary integral equation with the Stratton–Chu representation formula of Maxwell's equations, that is,

$$-\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \iint_{\Gamma} \left(\mu \frac{\dot{\mathbf{J}}(\mathbf{r}', \tau)}{4\pi R} - \frac{1}{\epsilon} \nabla \frac{\int_{-\infty}^{\tau} \nabla' \cdot \mathbf{J}(\mathbf{r}', \bar{t}) \mathrm{d}\bar{t}}{4\pi R} \right) \mathrm{d}\mathbf{r}' = -\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \mathbf{E}^i(\mathbf{r}, t) \tag{1}$$