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A system of fractional-order interval projection neural networks[☆]

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ABSTRACT

In this paper, a system of fractional-order interval projection neural networks is introduced and investigated. Under some suitable assumptions, the existence and uniqueness of the equilibrium point of this type of interval projection neural networks is proved. Moreover, α-exponential stability of this type of neural networks is obtained. Also, in the last section, we give two numerical examples to demonstrate the validity of our results.

1. Introduction

In this paper, we study a system of fractional-order interval projection neural networks (for short, FIPNN) in $R^n \times R^m$ as follows:

$$\begin{cases} {}_0^C D_t^\alpha x(t) = P_{K_1}[x(t) - \rho(Ax(t) + A^* y(t)) - \rho a] - x(t), \quad t \geq 0, \\ x(0) = x_0, \\ {}_0^C D_t^\alpha y(t) = P_{K_2}[y(t) - \lambda(By(t) + B^* x(t)) - \lambda b] - y(t), \quad t \geq 0, \\ y(0) = y_0, \end{cases} \tag{1.1}$$

where $0 < \alpha \leq 1$, P_{K_1} and P_{K_2} are two projection operators, ρ, λ are two positive constants, $a, x_0 \in R^n$ and $b, y_0 \in R^m$, and

$$\begin{cases} A \in A_I = \left\{ (a_{ij})_{n\times n} : \underline{A} \leq A \leq \overline{A}, \text{ i.e., } \underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij} \right\}, \\ A^* \in A_I^* = \left\{ (a_{ij}^*)_{n\times m} : \underline{A^*} \leq A^* \leq \overline{A^*}, \text{ i.e., } \underline{a^*}_{ij} \leq a_{ij}^* \leq \overline{a^*}_{ij} \right\}, \\ B \in B_I = \left\{ (b_{ij})_{m\times m} : \underline{B} \leq B \leq \overline{B}, \text{ i.e., } \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij} \right\}, \\ B^* \in B_I^* = \left\{ (b_{ij}^*)_{m\times n} : \underline{B^*} \leq B^* \leq \overline{B^*}, \text{ i.e., } \underline{b^*}_{ij} \leq b_{ij}^* \leq \overline{b^*}_{ij} \right\} \end{cases}$$

with the set K_1 and K_2 are two closed rectangles given by

$$K_1 = \left\{ x \in R^n : c_i \leq x_i \leq d_i, \; i = 1, 2, \ldots, n \right\}$$

and

$$K_2 = \left\{ y \in R^m : h_i \leq y_i \leq l_i, \; i = 1, 2, \ldots, m \right\},$$

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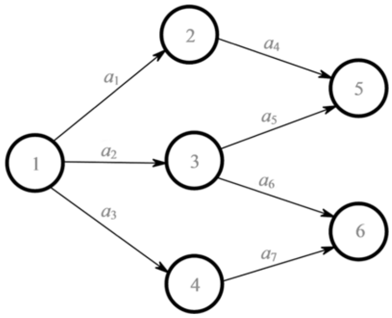

Fig. 1. 7-Arc 6-node traffic network.

where c_i, d_i, h_i and l_i are possibly infinite scalars satisfying

$$-\infty < c_i < d_i < +\infty, \quad i = 1, 2, \ldots, n \quad and \quad -\infty < h_i < l_i < +\infty, \quad i = 1, 2, \ldots, m.$$

It is obvious that FIPNN (1.1) has the following equivalent form

$$\begin{cases} {}_0^C D_t^\alpha x_i(t) = P_{K_{1,i}}\left[x_i(t) - \rho\left(\sum_{j=1}^{n} a_{ij}x_j(t) + \sum_{j=1}^{m} a_{ij}^* y_j(t)\right) - \rho a_i\right] - x_i(t), & t \geq 0, \\ x_i(0) = x_{i0}, \quad i = 1, 2, \ldots, n, \\ {}_0^C D_t^\alpha y_i(t) = P_{K_{2,i}}\left[y_i(t) - \lambda\left(\sum_{j=1}^{m} b_{ij}y_j(t) + \sum_{j=1}^{n} b_{ij}^* x_j(t)\right) - \lambda b_j\right] - y_i(t), & t \geq 0, \\ y_i(0) = y_{i0}, \quad i = 1, 2, \ldots, m, \end{cases} \tag{1.2}$$

where

$$\underline{a}_{ij} \leq a_{ij} \leq \overline{a}_{ij}, \qquad \underline{a^*}_{ij} \leq a_{ij}^* \leq \overline{a^*}_{ij}, \qquad \underline{b}_{ij} \leq b_{ij} \leq \overline{b}_{ij}, \qquad \underline{b^*}_{ij} \leq b_{ij}^* \leq \overline{b^*}_{ij}$$

with

$$K_{1,i} = \{x_i \in R : c_i \leq x_i \leq d_i\}, \quad i = 1, 2, \ldots, n$$

and

$$K_{2,i} = \{y_i \in R : h_i \leq y_i \leq l_i\}, \quad i = 1, 2, \ldots, m.$$

Some special cases of (1.1) have been studied in recent literature. For instance, if $\underline{A} = A = \overline{A}$, $\underline{B} = B = \overline{B}$, $\underline{A^*} = A^* = \overline{A^*} = 0$ and $\underline{B^*} = B^* = \overline{B^*} = 0$, then (1.1) reduces to two separate fractional projective dynamical system, which was first studied by Wu and Zou [1] in 2014. If $\alpha = 1$, $\underline{A^*} = A^* = \overline{A^*}$ and $\underline{B^*} = B^* = \overline{B^*}$, the interval projective dynamical system has been considered by Wu et al. [2] in 2013. Earlier special case of (1.1), a simple interval projection neural network was studied by Ding and Huang [3] in 2008. There are a number of applications and theoretical results regarding the projection neural networks, for details, we refer to [1–16] and the references therein. In particular, systems with fractional order have frequently appeared in the research area in the last two decades (see, for example, [1,17–24]). This paper considers a system with fractional order and interval constraints. We first illustrate how this type of system can be put into practice by restating an example that can be found in [6].

Example 1.1. Wardropian user equilibrium tatonnement model was constructed to consider day-to-day adjustments of flows and costs on general traffic network, as explained in detail by Friesz et al. [6]. For the sake of brevity, we discuss here the simple traffic network (7 arcs and 6 nodes) illustrated in Fig. 1. There is a single origin (node 1), two destinations (nodes 5, 6) and four paths. Path p_1 consists of arcs a_1 and a_4, path p_2 consists of arcs a_2 and a_5, path p_3 consists of arcs a_2 and a_6, path p_4 consists of arcs a_3 and a_7. In particular note the following:

- $[0, T]$: the closed time interval of interest;
- $u_{ij}(t)$: the travel cost reported by the advanced traveler information system (ATIS) for origin i and destination j on day t;
- $h_{p_n}(t)$: the flow on path p_n on day t, which is measured as the flow at the entrance of the first arc of path p_n on day t;
- $\gamma_{a_m p_n}$: the arc-path incidence matrix, specifically $\gamma_{a_m p_n} = 1$ if $a_m \in p_n$ and $\gamma_{a_m p_n} = 0$ otherwise;
- $f_{a_m}(t)$: the flow on arc a_m at time t; $f_{a_m}(t)$ and h_{p_n} are related by the identity $f_{a_m}(t) = \sum_{n=1}^{4} \gamma_{a_m p_n} h_{p_n}(t)$;
- $C_{a_m}(f_{a_m}(t))$: the unit cost of flow on arc a_m on day t;
- $T_{ij}(u_{ij}(t))$: the travel demand between origin i and destination j on day t.