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Compact finite difference modeling of 2-D acoustic wave propagation

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HIGHLIGHTS

- A new application of mimetic fourth-order finite differences coupled to leap-frog time integration to wave propagation phenomena.
- A new 2-D compact formulation on nodal meshes with Crank–Nicolson time integration.
- A new mimetic formulation based on leap-frog scheme and Crank–Nicolson time integration on a 2-D staggered grids.
- Two new fourth-order compact finite difference schemes to model acoustic motion under homogeneous boundary conditions.

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ABSTRACT

We present two fourth-order compact finite difference (CFD) discretizations of the velocity–pressure formulation of the acoustic wave equation in 2-D rectangular grids. The first method uses standard implicit CFD on nodal meshes and requires solving tridiagonal linear systems along each grid line, while the second scheme employs a novel set of mimetic CFD operators for explicit differentiation on staggered grids. Both schemes share a Crank–Nicolson time integration decoupled by the Peaceman–Rachford splitting technique to update discrete fields by alternating the coordinate direction of CFD differentiation (ADI-like iterations). For comparison purposes, we also implement a spatially fourth-order FD scheme using non compact staggered mimetic operators in combination to second-order leap-frog time discretization. We apply these three schemes to model acoustic motion under homogeneous boundary conditions and compare their experimental convergence and execution times, as grid is successively refined. Both CFD schemes show four-order convergence, with a slight superiority of the mimetic version, that leads to more accurate results on fine grids. Conversely, the mimetic leap-frog method only achieves quadratic convergence and shows similar accuracy to CFD results exclusively on coarse grids. We finally observe that computation times of nodal CFD simulations are between four and five times higher than those spent by the mimetic CFD scheme with similar grid size. This significant performance difference is attributed to solving those embedded linear systems inherent to implicit CFD.

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1. Introduction

Wave motion in an acoustic medium with density ρ and adiabatic compression modulus k can be modeled by the system

$$\begin{cases} \dfrac{1}{k}\dfrac{\partial u}{\partial t} = -\nabla \cdot \vec{v} + f, \\ \rho \dfrac{\partial \vec{v}}{\partial t} = -\nabla u \end{cases} \tag{1}$$

where dependent variables correspond to the particle velocity vector $\vec{v} = (v, w)$ and the pressure field u. This velocity–pressure formulation allows stating any consistent combination of free-surface or rigid-wall boundary conditions on the appropriate physical variable. Current finite difference (FD) methods for this model are mostly implemented on spatial staggered grids, and use explicit time discretization performed by short second-order stencils, in order to limit memory computer requirements. Applications cover from seismic imaging of Earth interior [1,2], visualization of sound fields [3], and seismic motion in marine scenarios [4,5]. In a staggered grid, wave fields and material parameters are defined at intermediate grid positions, in such a way that an unknown field is located at the center of those it depends. Numerical differentiation exploits this geometrical distribution that halves the grid spacing to gain accuracy. FD staggered-grid modeling of wave propagation on heterogeneous media having drastic variation of material properties has exhibited minimal dispersion and numerical anisotropy in the more general case of elastic rheologies [6]. These numerical techniques can achieve further accuracy by employing large computational stencils with fourth or even higher order. Now, this practice might require of more time-consuming domain decomposition procedures in parallel FD applications, because of the higher demand of data interchange. In addition, high order FD discretization of Neumann boundary conditions based on lateral stencils can degenerate in numerical instabilities on wave propagation problems [7–9].

As an alternative, implicit compact FD methods can accomplish high-order accuracy by solving for the spatial derivatives, a linear system along each gridline. In a compact FD approximation, the discrete differential values at subsequent grid points are coupled, and the difference formula requires fewer grid points than those used by explicit FD stencils. A general family of compact schemes have attained fast implementations after including the alternating direction implicit (ADI) methodology. ADI methods were introduced by Peaceman, Rachford, and Douglas [10,11] for 2-D diffusion problems in infinite domains, and their formulation is unconditionally stable and well suited for efficient implementations. These pioneer works have been followed by emerging formulations of higher order compact-ADI FD with moderate computational cost, and some recent contributions are [12–15]. At the implementation level, the main drawback of these strategies is the simultaneous approximation of derivate values at all points along a grid line by solving a linear (usually tridiagonal) system. Although, this problem can be solved cheaply and precisely by efficient methods based on Thomas' algorithm, as an effort to develop conservative FD, Castillo and collaborators have proposed the mimetic differentiation formulas on staggered grids that provide second-, fourth-, and sixth-order accuracy at all grid locations including boundaries [16,17]. Among several applications of mimetic FD, fourth-order modeling of surface waves and earthquake ruptures on elastic media count as related to methods presented in this paper [18,9]. Recently, Abouali and Castillo introduce a compact factorization of the high-order mimetic operators in terms of the second-order ones and additional auxiliary operators [19]. Both latter operators exhibit shorter stencils compared to the ones provided by the former, and represent compact choices for explicit differentiation.

In this paper, we implement and compare three numerical methods to solve the velocity–pressure acoustic system on rectangular domains. All of them present fourth-order spatial differentiation. The first method uses implicit compact FD on nodal grids similar to those proposed by [20,21], in combination to a Crank–Nicolson time integration efficiently solved by a Peaceman–Rachford ADI decomposition. The second method shares same time-stepping strategy, but applies explicit compact mimetic FD to reduce computer execution times. This scheme represents the first 2-D application of such differentiation operators. Both compact formulations are detailed in Section 2 of this paper. Section 3 briefly describes our last scheme based on non-compact mimetic differentiation coupled to a second-order leap-frog discretization of time derivatives. This third numerical method falls into the family of modern space–time staggered FD methods on wave propagation, and its results as used as a reference. Section 4 presents numerical solutions to a test case with homogeneous Dirichlet boundary conditions, and compares accuracy and convergence achieved by these three methods. Finally, Section 5 summarizes our conclusions and points out some extensions of this work.

2. Formulation of compact finite difference (CFD) methods

In this section, we revise standard fourth-order operators for compact finite differentiation of a smooth field given discretely on a 1-D nodal grid, and use them to formulate a numerical scheme for the 2-D acoustic model stated by Eq. (1). Without any loss of generality, we assume as problem domain $\Omega = [0, 1] \times [0, 1]$, and a Dirichlet boundary condition $u(x, y, t) = u_0(x, y, t)$ for $(x, y) \in \partial\Omega$. Next, we present a novel method for this model based on the mimetic compact CFD operators on 1-D staggered grids proposed by Abouali and Castillo in [19] that explicitly carry out the differentiation process without solving any linear system as required by the former implicit CFD on nodal grids. We emphasize this implementation advantage on some final comparative comments on both CFD methods. The benefit of the explicit nature of this new mimetic CFD scheme is later evidenced on reported CPU consumption times in our numerical tests and given in a section below.