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Importance of components in k-out-of-n system with components having random weights

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ABSTRACT

The purpose of this paper is to study the component importance in k-out-of-n system with components having random weights. The system works if at least k components work and the total weight of all working components is above the required level c. A new measure of component importance, called *weighted importance*, for such a system is introduced and its relation with Birnbaum reliability measure of importance is studied. It is shown that the larger weight stochastically implies the larger weighted measure of importance. It is also shown that if two components are identically distributed then the larger weight stochastically implies the larger Birnbaum reliability measure of importance. Finally, it is proved that if the weight of a component is stochastically increased, then the weighted and Birnbaum reliability measures of importance are increased accordingly.

1. Introduction

Let $X_i, i = 1, \ldots, n$ be a set of independent Bernoulli random variables such that $p_i = P(X_i = 1)$ and $W_i, i = 1, \ldots, n$ be a set of independent random variables with finite supports $[a_i, b_i]$ where $a_i, b_i \in \mathbb{R}^+$. Let us also assume that W_i's and X_i's are independent. Consider a reliability system consisting of n components which functions at level c if at least k out of n components function and $\sum_{i=1}^n W_iX_i \geq c$, where W_i and X_i, respectively, denote the random weight and the state of component i. In other words, let $\mathbf{X} \equiv (X_1, \ldots, X_n)$, $\mathbf{W} \equiv (W_1, \ldots, W_n)$ and $\psi(\mathbf{X}, \mathbf{W})$ be the performance level of the system which is the total weight of at least k working components. Then

$$\psi(\mathbf{X}, \mathbf{W}) \geq c \ \text{(working state)} \Longleftrightarrow \sum_{i=1}^{n} X_i \geq k, \quad \sum_{i=1}^{n} W_iX_i \geq c. \tag{1}$$

That is, in addition to the number of working components, the total weight of working components plays an important role in identifying the state of the system. This system was introduced in [1] and called *k-out-of-n system with components having random weights*. Let $(i_1, \ldots, i_n)$ be a permutation of (1, …, n) such that $a_{i_1} \leq a_{i_2} \leq \ldots \leq a_{i_n}$. As mentioned by Eryilmaz [1], the suitable values of c are in the set $[c_{\min}, c_{\max}] = [\sum_{j=1}^k a_{i_j}, \sum_{j=1}^n b_j]$.

Hereafter, we denote the k-out-of-n system with components having random weights by r-w-k-*out-of-n*. There are some examples that can be considered as r-w-k-out-of-n systems.

Power supply: Suppose there are n power plants in a state that plant i supply a random amount of power during a period of time, denoted by W_i. Now, if at least k number of plants are active and $\sum_{i=1}^n W_iX_i \geq c$ then, the system would be in a good

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condition where c is the required energy that is expected to be supplied by the system. Power plant might be a kind of wind farm or wind park which is a group of wind turbines in the same location used to produce energy.

Auditorium lighting system: Consider an auditorium lighting system which is composed of n circles of bulbs in the ceiling. Circle i consists of n_i bulbs in the parallel structure, with a switch to turn on, $i = 1, \dots, n$. The random weight of switch i is the light coming from circle i which can be measured by the number of functioning bulbs in the circle. It is obvious that the amount of light in the auditorium system depends on the number of working switches and their random weights.

Queueing system [1]: Consider a batch service queueing system in which the server can take k customers into the service. If there are less than k customers in the queue, the servers wait until the queue size reaches k. Assume that after the batch of k customers were taken by the system each customer is served by one of the n servers. Thus, at least k of n servers must be operational to serve simultaneously the customers. Now, suppose that the performance of the system is satisfactory in a given day if at least c customers served. This is possible if at least k servers operate and $\sum_{i=1}^{n} W_i X_i \geq c$ where W_i denotes the number of customers served by server i, $i = 1, \dots, n$.

Eryilmaz [1] investigated the reliability of the system introduced in (1) when W_i's are non-zero discrete random variables with support $[a_i, b_i]$ and presented a recursive formula for computing the system state probabilities. In particular, when $W_i \equiv 1, i = 1, \dots, n$ and $c = k$, the system is reduced to the regular k-out-of-n system which is the most important coherent system in the reliability context and has been extended to various models (see, for example, [2]). Further, if $W_i \equiv w_i, i = 1, \dots, n$ and $k = 1$, the system is reduced to weighted-c-out-of-n system, which was introduced in [3] and studied by Higashiyama [4], Chen and Yang [5], Samaniego and Shaked [6] and Eryilmaz and Bozbulut [7]. If $k = 1$, the system is a weighted-c-out-of-n system with components having random weights.

The rank of components with respect to their importance to the state of the system is of practical importance in the reliability context. The importance of components, in a system consisting of non-weighted components, depends on the position of components in the system and the reliability or the lifetime distribution of components. Thus, the importance measures can be classified as structural importance, reliability importance and lifetime importance measures (cf. [8–10]). Structural importance measures are defined based on the importance of the position of components in some senses and used when only the structure function of the system is known. When both the system structure and the reliability of components are available, reliability importance measures are defined. The most common structural and reliability measures of importance are the measures proposed by Birnbaum [8]. Similarly, lifetime importance measures depend on both the system structure and the lifetime distribution of components. A variety of importance measures have been defined and used in the reliability literature, such as [8,11–21]. For a comprehensive survey of measures of importance, see [22,9,10].

In the weighted systems, in addition to the position and reliability/lifetime distribution, the weight has the effect on the component importance. Eryilmaz and Bozbulut [7] studied the importance of components in a weighted-c-out-of-n system with respect to Birnbaum marginal and joint reliability importance measures and Barlow–Proschan lifetime importance measure. They used numerical calculations to show that the ranking of the importance of components in a weighted-c-out-of-n system depends on the weight of components in addition to their reliabilities or lifetime distributions. It is obvious that for computing Birnbaum reliability measures and Barlow–Proschan lifetime measure, the reliability and the lifetime distribution of components are required to be known.

The main purpose of this paper is to consider and study components importance of the *r-w-k-out-of-n* system. In such a system, structural importance of all components are equal due to k-out-of-n structure. Now, suppose that the reliability/lifetime distribution of the components are not known. Therefore, with this set-up, to measure the importance of the components we need to define a measure that depends only on the distribution of component weights. In Section 2, we propose a new measure of importance called *weighted importance (WI) measure* for *r-w-k-out-of-n* system and investigate its properties. We also study the properties of Birnbaum reliability importance (BRI) measure for *r-w-k-out-of-n* system and its relation with the WI measure. By an example, we show that the ranking of components of an *r-w-k-out-of-n* systems with respect to the WI and BRI measures depends on k and c. Section 3 is devoted to comparisons of components importance according to the usual stochastic order. Summary and conclusions of the paper are given in Section 4.

We end this section by introducing BRI measure that we use later in the paper. Consider a binary coherent system with n independent binary components and let X_i denote the state of component i which $X_i = 1$ if the component functions for the desired time and $X_i = 0$ if the component fails during this time. Let ϕ be the structure function of the system and $h(\mathbf{p}) = E[\phi(\mathbf{X})]$ be the reliability of the system where $\mathbf{X} = (X_1, \dots, X_n)$, $\mathbf{p} = (p_1, \dots, p_n)$ is the reliability vector of components and E stands for the expected value. BRI measure of component i (denoted by $I_R^B(i)$) is defined as

$$\begin{aligned} I_R^B(i) &= P(\phi(\mathbf{X}) = 1 | X_i = 1) - P(\phi(\mathbf{X}) = 1 | X_i = 0) \\ &= \frac{\partial}{\partial p_i} h(\mathbf{p}). \end{aligned}$$

2. Importance measures of components

In this section we investigate the components importance in the *r-w-k-out-of-n* system. In many situations, the component reliabilities of the system are not available. Therefore, to investigate the importance of a component in an *r-w-k-out-of-n* system, we need a measure of importance only in terms of the component weights which is defined next.