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Neuroelectric source localization by random spatial sampling

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ABSTRACT

The magnetoencephalography (MEG) aims at reconstructing the unknown neuroelectric activity in the brain from the measurements of the neuromagnetic field in the outer space. The localization of neuroelectric sources from MEG data results in an ill-posed and ill-conditioned inverse problem that requires regularization techniques to be solved. In this paper we propose a new inversion method based on random spatial sampling that is suitable to localize focal neuroelectric sources. The method is fast, efficient and requires little memory storage. Moreover, the numerical tests show that the random sampling method has a high spatial resolution even in the case of deep source localization from noisy magnetic data.

1. Introduction

Magnetoencephalography (MEG) [1] is a completely non-invasive imaging technique to map the neuroelectric activity from the measurements of the magnetic field that the activity itself induces outside the head. Due to its high temporal resolution – in the millisecond scale [2] – MEG is particularly attractive for mapping fast cerebral responses to spontaneous and/or evoked stimuli. From the analysis of the temporal evolution of the measured magnetic field distribution we can infer just partial information on the localization of active brain regions. In order to better focus neuroelectric sources, we have to solve the neuroelectric inverse problem aiming at reconstructing the neuronal current image once a measured magnetic field distribution outside the head is given.

Since the magnetic field decreases very fast as the distance between the electric sources and the sensor sites increases, the measured magnetic field may be very weak. For this reason MEG magnetometers are equipped with SQUIDs (Superconducting Quantum Interference Devices), which are very sensitive detectors of the magnetic flux [1]. Moreover, MEG measurements are affected by high noise due to electromagnetic sources in the external environment and to the bioelectric activity generated by the muscular activity of the patient himself. Usually, these disturbances generate a magnetic signal of strength comparable with the signal of interest.

Another challenge of MEG is in its ill-conditioned nature; in fact, the radial – w.r.t. the inner skull – component of the neuroelectric current does not produce any magnetic field in the outer space and cannot be detected. This means that a single measured field could be generated by an infinite number of current distributions and further assumptions could be made in order to force the inverse problem to have a unique solution [3].

When solving the MEG inverse problem, we are interested in reconstructing the neuroelectric current image with high accuracy – in the order of few millimeters – having available only few magnetic data—usually, a few hundreds. Thus, the MEG inverse problem can be seen as an inverse problem with incomplete data. On the other hand, neurophysiologic studies

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have put in evidence that the neuroelectric current distribution is localized in small regions of the brain, i.e. the neuroelectric current distribution is spatially sparse. As a consequence, it is reasonable to expect that only few elementary sources might be sufficient to characterize and reconstruct the unknown current vector [4–6].

After these observations, in this paper we propose a new method based on random sampling, suitable to solve the MEG inverse problem under the sparsity assumption. The key ingredient of the method lies in the fact that we represent the electric current distribution we want to reconstruct by a sample ensemble of few *localized* elementary sources. Under the sparsity assumption, just few elementary sources, randomly chosen from a large dictionary, are sufficient to well reconstruct the unknown current distribution. Some first results on the solution of the MEG inverse problem by the random sampling method can be found in [7] where it is shown that random sampling reduces significantly the ill-conditioning of the inverse problem so acting as a regularization technique. Moreover, the algorithm requires little memory storage and is very fast. Here, we deal with the problem of localizing focal deep sources from noisy magnetic data and show that the random sampling method combined with a shrinkage method has a high spatial resolution so that it can be effectively used in neuroimaging applications.

The paper is organized as follows. In Section 2, we recall the model usually used to describe the MEG forward problem and set the MEG inverse problem. In Section 3 we describe the random sampling method for the solution of the MEG inverse problem. Section 4 is devoted to several numerical tests showing the good performances of the proposed method. Finally, Section 5 contains some comments and conclusions.

2. The MEG forward and inverse problems

Following the classical model by Geselowitz [8,9], we describe the head as a conductor consisting of homogeneous, nested, non intersecting regions, $V_i,\ i = 0, \dots, m$, each one having constant conductivity, σ_i. In the following we assume that the neuroelectric current flows just inside the innermost region V_0, which represents the brain. From the quasi-static Maxwell's equations, it follows that the electric current density $\mathbf{J}(\mathbf{r})$ flowing in V_0, and the external magnetic field $\mathbf{B}(\mathbf{r})$, with $\mathbf{r}$ outside V_m, are related by the *Biot–Savart law*

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{V_0} \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}\, d\mathbf{r}', \tag{2.1}$$

where μ_0 is the magnetic permeability in the vacuum.

The magnetometers are located in N sites, $\mathbf{q}_i,\ i = 1, \dots, N$, that belong to a surface Σ external to the head. Each magnetometer measures the magnetic field along the direction $\mathbf{e}(\mathbf{q}_i)$, which is the normal w.r.t. Σ in $\mathbf{q}_i$. Now, let $\mathcal{B}_{\mathbf{e}}(\mathbf{q}_i, \mathbf{J}) := \mathbf{B}(\mathbf{q}_i) \cdot \mathbf{e}(\mathbf{q}_i)$ be the integral operator relating the neuroelectric current and the magnetic field it generates in $\mathbf{q}_i$, projected along $\mathbf{e}(\mathbf{q}_i)$. Recalling that for any three vectors in $\mathbb{R}^3$ it holds $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{z} = -(\mathbf{z} \times \mathbf{w}) \cdot \mathbf{v}$, we obtain the relation

$$\mathcal{B}_{\mathbf{e}}(\mathbf{q}_i, \mathbf{J}) = \frac{\mu_0}{4\pi} \int_{V_0} \left(\mathbf{e}(\mathbf{q}_i) \times \frac{\mathbf{r}' - \mathbf{q}_i}{|\mathbf{r}' - \mathbf{q}_i|^3} \right) \cdot \mathbf{J}(\mathbf{r}')\, d\mathbf{r}', \tag{2.2}$$

which is linear w.r.t. $\mathbf{J}$ (here, $\mathbf{v} \times \mathbf{w}$ and $\mathbf{v} \cdot \mathbf{w}$ are the usual cross and scalar products of vectors in $\mathbb{R}^3$, respectively, and $|\mathbf{v}|$ is the Euclidean norm).

In a realistic head geometry the forward MEG problem cannot be solved analytically, therefore numerical methods are needed. Usually, to solve numerically the forward problem, Boundary Element Method, Finite Element Method or Finite Difference Method are used [10,11]. All these methods require a large number of computational points to achieve high spatial resolution so that they both require high memory storage and have high computational load.

Having at hand the forward model, we can set the MEG inverse problem. This consists in estimating the neuroelectric current distribution $\mathbf{J}$ from the measurements of the external magnetic field, $G_i,\ i = 1, \dots, N$. Therefore, the MEG inverse problem lies in minimizing the discrepancy

$$\Delta(\mathbf{J}) = \sum_{i=1}^{N} (G_i - \mathcal{B}_{\mathbf{e}}(\mathbf{q}_i, \mathbf{J}))^2, \tag{2.3}$$

w.r.t. the current distribution $\mathbf{J}$, once the measurements $G_i,\ i = 1, \dots, N$, are given. Since the integral operator (2.2) has a non-trivial kernel, additional constraints, coming from the physics of the problem, have to be added so that the inverse problem has a unique solution [12,3]. This a priori information must be included into the inversion method to produce a physically meaningful solution. Our aim is to use sparsity assumption and random sampling to reduce the dimensionality of the inverse problem and, at the same time, its ill-conditioning.

3. The random sampling method

To solve the inverse problem we model the total current as a sum of a finite number of *elementary sources*, i.e.

$$\mathbf{J}(\mathbf{r}) \approx \sum_{k=1}^{M} \mathbf{J}_k\, \psi_k(\mathbf{r}), \tag{3.1}$$