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Optimal investment and proportional reinsurance for a jump–diffusion risk model with constrained control variables

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ABSTRACT

This paper considers the optimal control problem with constraints for an insurer. The risk process is assumed to be a jump–diffusion process, and the risk can be reduced through a proportional reinsurance. In addition, the surplus can be invested in the financial market consists of one risk-free asset and one risky asset. The diffusion term can explain the uncertainty associated with the surplus of the insurer or the additional small claims. The objective of the insurer is to maximize the expected exponential utility of terminal wealth. This optimization problem is studied in two cases depending on the diffusion term's explanations. In all cases, with normal constraints on the control variables, the value functions and the corresponding optimal strategies are given in a closed form. Numerical simulations are presented to illustrate the effects of parameters on the optimal strategies as well as the economic meaning behind.

1. Introduction

Nowadays, insurance companies have become major institutions in worldwide financial markets. They are actively involved in trading activities in various financial markets. Consequently, the optimal asset allocation problem is important for insurance companies. For the optimal asset allocation problem, the key difference between insurance companies and their financial counterparts is the presence of insurance liabilities, which mainly depend on insurance claims. In the fierce competition among businesses, insurance companies are trying to take various strategies to increase their reserve and then minimize their risk. In general, insurance companies face two sources of risks: a risk arising from insurance claims and a market risk arising from risky investments in financial markets. Reinsurance is one of the ways by which insurance companies effectively transfer parts of their risks arising from insurance claims. In order to reduce the market risk, insurance companies tend to invest in some risk-free assets such as short-term bonds and money market funds. Therefore, the optimal investment and reinsurance problems with various objectives in insurance risk management have attracted a lot of attention in the past few years, and a significant amount of works have been done on this topic.

Some scholars focus on maximizing the expected utility of the insurers' terminal wealth. For example, Browne [1] obtains an optimal investment strategy for an insurer whose surplus process is modeled by a drifted Brownian motion and who is allowed to invest in a risky asset with the price governed by a geometric Brownian motion (GBM). Irgens and Paulsen [2] consider the optimal reinsurance and investment problem for an insurer whose surplus follows a classical risk process

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perturbed by a diffusion, where the compound Poisson process stands for the large claims and the diffusion term represents the additional small claims (A-C, for short). Aiming at maximizing expected utility, they find the optimal strategies for three different utility functions. Yang and Zhang [3] study the optimal investment problem for an insurer with a jump–diffusion surplus process, in which the diffusion term stands for the uncertainty associated with the surplus of the insurer (U-S, for short). Wang [4] extends the results of Yang and Zhang [3] to the case of multiple risky assets. Liang and Guo [5] consider the optimal combining quota-share and excess-of-loss reinsurance problem, they derive the closed form expressions of the optimal strategies and value function. Under the assumption that the instantaneous rate of investment return follows an Ornstein–Uhlenbeck process, the authors study the optimal investment and proportional reinsurance problem in Liang et al. [6]. Liang et al. [7] investigate the optimal proportional reinsurance and investment problem for a jump–diffusion risk model in a constant elasticity of variance stock market. In addition, there are some scholars paying their attention to other optimization objectives. We refer readers to Bauerle [8], Bai and Zhang [9], Bi and Guo [10] for mean–variance criterion, Schmidli [11,12], Promislow and Young [13] for ruin probability criterion, Hald and Schmidli [14] and Liang and Guo [15,16] for adjustment coefficient criterion.

The constraints on the controls of many of the above models are not natural, and some of them are not even legally permitted. In the financial market, the short-selling constraint is one of the main factors which make models more realistic. Since countries such as China impose restrictions on short-selling, it is important to study the associated optimal control problems without short selling opportunity. Under short-selling constraint, Bai and Guo [17] discuss the optimal proportional reinsurance and investment problem with multiple risky assets for a drifted Brownian motion risk model. Xu et al. [18] consider the same problem as Bai and Guo [17], while the surplus process is described by a perturbed classical risk model and the return of the risky asset is driven by a drifted Brownian motion plus a compound Poisson process. Bai and Guo [19] study the problem of Bai and Guo [17] again with the assumption that the insurer purchases excess-of-loss reinsurance. Suppose that the claim process is modeled by a jump–diffusion process and the price of risky assets is given by a GBM, Liu and Zhang [20] investigate the problem of optimal investment and excess-of-loss reinsurance with short-selling constraint. Zhang et al. [21] solve the problem of optimal investment and reinsurance under no short-selling and no borrowing. Generally speaking, both the insurer and the reinsurer are required to satisfy the net profit conditions. Therefore, the reinsurance strategy such that insurance company transfers all of the risks due to insurance claims to reinsurance company should be prohibited. Liang et al. [22] first consider the net profit condition in the optimal reinsurance and investment problem. There are very few results concerning the optimal control problem for the insurer with constraints both on the investment strategy and the reinsurance strategy.

Motivated by the work of Liang et al. [22], in this paper, we study the optimal proportional reinsurance and investment problem for a jump–diffusion risk model under the normal constraints on the control variables. There are two kinds of different explanations about the diffusion term in the jump–diffusion risk model. One is the A-C case as described in Irgens and Paulsen [2], the other is the U-S case is given as in Yang and Zhang [3]. Close-form expressions for the value functions and optimal control strategies are obtained in both cases. By comparing the results, we find that, although the insurance company pays the same premium for the reinsurance company in both cases, there is a great difference between the optimal strategies for those two cases. The optimal investment strategy and the optimal reinsurance strategy do not affect each other in the U-S case. However, the optimal investment strategy is affected by the optimal reinsurance strategy and vice versa in the A-C case.

This paper proceeds as follows. We formulated the model assumptions in Section 2. In Sections 3 and 4, we discuss the optimal investment and reinsurance strategy for the U-S case as well as the A-C case. The closed-form expressions for the optimal strategies and the corresponding value functions are derived. In Section 5, numerical simulations are presented to illustrate our results. Section 6 concludes this paper.

2. Model formulation

In this section, we will give models and some basic assumptions. We assume that trading in the reinsurance and financial markets is continuous, without taxes or transaction costs, and that all assets are infinitely divisible. We define a complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\in[0,T]}, P)$ satisfying the usual condition. $\{\mathscr{F}_t\}_{t\in[0,T]}$ is an augmented filtration generated by a compound Poisson process $\{S_t\}$ and two Brownian motions $\{W_t^{(1)}\}$ and $\{W_t^{(2)}\}$ with $\mathscr{F} = \mathscr{F}_T$, where T is a fixed and finite time horizon.

If both reinsurance and investment are absent, the insurer's surplus process is described by a jump–diffusion risk model, in which the surplus X_t of the insurer at time t is

$$X_t = x + ct + \beta W_t^{(1)} - \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0, \tag{2.1}$$

where x is the initial surplus, $c > 0$ is the premium rate, $\beta \geq 0$ is a constant, $\{W_t^{(1)},\ t \geq 0\}$ is a standard Brownian motion, $S_t := \sum_{i=1}^{N(t)} Y_i$ is a compound Poisson process, where $\{N(t),\ t \geq 0\}$ is a homogeneous Poisson process with intensity λ, and $\{Y_i,\ i \geq 1\}$ is a sequence of positive independent and identically distributed random variables with common distribution $F(y)$, mean value $\mu = E[Y_i]$, and moment generating function $M_Y(s) = E[e^{sY_i}]$. We assume that $E[Ye^{sY}] = M_Y'(s)$ exists for