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A new family of Marshall–Olkin extended generalized linear exponential distribution

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A B S T R A C T

The purpose of this paper is to introduce a new family of Marshall–Olkin extended generalized linear exponential distribution. This new family has the advantage of being capable of modeling various shapes of aging and failure criteria. The proposed family includes as special cases several Marshall–Olkin extended distributions studied in the literature such as exponential, Rayleigh, linear failure rate and Weibull, among others. Some statistical and reliability properties of the new family are discussed and an explicit expressions for the quantiles are derived. The method of the maximum likelihood estimation is used to estimate the unknown parameters. In addition, the asymptotic confidence intervals for the parameters are derived from the Fisher information matrix. Finally, the obtained results are validated using some real data sets and it is shown that the new family provides a better fit than some other known distributions.

1. Introduction

Any statistical analysis depends greatly on the statistical model used to represent the phenomena under study. Hence, the larger the class of statistical models available to the statistician the easier it is to choose a model. A quick survey of the models in common use reveals the abundance of statistical models in the literature. However, data of many important and practical problems do not follow any of the probability models available. In such cases a non-parametric model may be recommended. Although a two parameter distribution may provide reasonably precision in fitting data, it may be still desirable to extend the flexibility of any distribution to allow for better description of data without having to resort to non-parametric models. Since there is a clear need for extended forms of these distributions, a significant progress has been made toward the generalization of some well-known distributions and their successful applications to problems in areas such as engineering, finance, economics and biomedical sciences, among others. An interesting idea of generalizing a distribution, known in the literature as Marshall and Olkin (M–O) extended distribution. In [1], a new method of adding a parameter into a family of distributions was introduced and studied. The resulting distribution, known as M–O extended distribution, includes the baseline distribution as a special case and gives more flexibility to model various types of data. According to [1], if $\overline{F}(x)$ denotes the survival function (sf) of a continuous random variable X, then the timely honored device of adding a new parameter results in another sf $\overline{G}(x)$ defined by

$$\overline{G}\,(x;\alpha) = \frac{\alpha\overline{F}\,(x)}{1-\overline{\alpha}\overline{F}\,(x)}, \quad -\infty \le x \le \infty,\ \alpha \ge 0,\ \overline{\alpha} = 1-\alpha. \tag{1.1}$$

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The M–O extended distributions offer a wide range of behavior than the basic distributions from which they are derived. The property that the extended form of distributions have an interesting hazard function depending on the value of the added parameter and therefore can be used to model real situations in a better manner than the basic distribution (cf. [1–13]).

Let $h(x)$ and $r(x)$ denote the hazard rate functions of the transformed distribution and the original distribution, respectively. Marshall and Olkin [1] have called the additional shape parameter "tilt parameter", since the hazard rate of the new family is shifted below ($\alpha > 1$) or above ($0 < \alpha \leq 1$) the hazard rate of the underlying distribution, that is, for all $x \geq 0$, $h(x) \leq r(x)$ when $\alpha > 1$, and $h(x) \geq r(x)$ when $0 < \alpha \leq 1$.

Marshall and Olkin [1] introduced a two-parameter extension of the exponential [M–OEE (α, λ)] distribution with sf

$$\overline{G}(x; \alpha, \lambda) = \frac{\alpha}{e^{\lambda x} - \overline{\alpha}}, \quad x > 0, \ \alpha, \lambda \geq 0;$$

three-parameter extension of the Weibull distribution [M–OEW (α, λ, β)] with sf

$$\overline{G}(x; \alpha, \lambda, \beta) = \frac{\alpha e^{-\lambda x^{\beta}}}{1 - \overline{\alpha} e^{-\lambda x^{\beta}}}, \quad x > 0, \ \alpha, \lambda, \beta > 0,$$

and two parameter extension of the Rayleigh distribution [M–OER (α, θ)] with sf

$$\overline{G}(x; \alpha, \theta) = \frac{\alpha e^{-\theta x^2}}{1 - \overline{\alpha} e^{-\theta x^2}}, \quad x > 0, \ \alpha, \theta > 0.$$

In addition, based on M–O method, Ghitany and Kotz [2] introduced and studied extension of the linear failure rate distribution [M–OELFR $(\alpha, \theta, \lambda)$] with sf

$$\overline{G}(x; \alpha, \theta, \lambda) = \frac{\alpha \exp\left\{-\left[\theta x^2 + \lambda x\right]\right\}}{1 - \overline{\alpha} \exp\left\{-\left[\theta x^2 + \lambda x\right]\right\}}, \quad x > 0, \ \alpha, \lambda, \beta > 0.$$

Recently, Mahmoud and Alam [14] introduced and studied a new interesting model called generalized linear exponential (GLE) distribution with parameters θ, λ and β with sf

$$\overline{F}(x; \theta, \lambda, \beta) = \exp\left\{-\left[\frac{\theta}{2}x^2 + \lambda x\right]^{\beta}\right\}, \quad x \geq 0, \ \theta, \lambda, \beta > 0. \tag{1.2}$$

All the M–O distributions mentioned above and the GLE distribution are special cases of the new family that is introduced and studied in Section 2. In Section 3, some statistical and reliability properties of the new family are discussed. In Section 4, the method of maximum likelihood estimation is used to estimate the unknown parameters. In addition, simulation is utilized to calculate the unknown parameters and to study their properties. Section 5 gives some applications to explain how some real data sets can be modeled by the new family. Finally, in Section 6, some conclusions and remarks of the current and future research are presented.

2. The new model

In this section, we propose the M–O generalized linear exponential (M–OGLE) distribution. We derive density, survival, hazard rate and reversed hazard rate functions of the new family. The proofs of theorems, propositions and lemmas are deferred to Appendix.

2.1. M–OGLE specification

Let $\Theta = (\theta, \lambda, \beta, \alpha)$ and substituting (1.2) in (1.1), we get a new distribution denoted as M–OGLE $(x; \Theta)$ distribution with sf

$$\overline{G}(x; \Theta) = \frac{\alpha\, \xi}{1 - \overline{\alpha}\, \xi}, \quad x \geq 0, \ \Theta > 0,$$

where $\xi = \exp -\left\{\left[\frac{\theta}{2}x^2 + \lambda x\right]^{\beta}\right\}$.

The corresponding cumulative distribution function (cdf) and the probability density function (pdf) are obtained, respectively as

$$G(x; \Theta) = \frac{1 - \xi}{1 - \overline{\alpha}\xi}, \quad x \geq 0, \ \Theta > 0,$$