Journal of Computational and Applied Mathematics 296 (2016) 724–738

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

The backward substitution method for multipoint problems with linear Volterra–Fredholm integro-differential equations of the neutral type

S.Yu. Reutskiy *

College of Mechanics and Materials, Hohai University, Nanjing 210098, PR China
State Institution "Institute of Technical Problems of Magnetism of the National Academy of Sciences of Ukraine", Industrialnaya St., 19, 61106, Kharkov, Ukraine

ARTICLE INFO

Article history:
Received 8 March 2015
Received in revised form 5 August 2015

Keywords:
Integro-differential equation
Volterra–Fredholm
Linear functional argument
Neutral type
Multipoint problem

ABSTRACT

The paper presents a new numerical method for solving multipoint boundary value problems for Volterra–Fredholm integro-differential equations with linear functional arguments. The method consists of replacing the initial equation by an approximate equation which has an exact analytic solution with a set of free parameters. These free parameters are determined by the use of the collocation procedure. Some examples are given to demonstrate the validity and applicability of the new method and a comparison is made with the existing results. Numerical results show that the proposed method is of high accuracy and is efficient for solving a wide class of the functional integro-differential equations of the most general form.

1. Introduction

In this paper we consider linear Volterra–Fredholm integro-differential equations (VFIDE) with linear functional arguments of the form:

$$\alpha(x)\, u^{(n)}(x) = \sum_{k=0}^{n-1} \beta_k(x)\, u^{(k)}(x) + \sum_{k=0}^{n} \gamma_k(x)\, u^{(k)}(q_k x + p_k) + \sum_{k=0}^{n} \int_a^{a_k x + b_k} K_{V,k}(x,t)\, u^{(k)}(t)\, dt + \sum_{k=0}^{n} \int_a^{b} K_{F,k}(x,t)\, u^{(k)}(t)\, dt + f(x). \tag{1}$$

The equation is subjected to the multipoint boundary conditions:

$$\sum_{k=0}^{n-1} \sum_{l=1}^{L} A_{j,k,l} u^{(k)}(\delta_l) = d_j, \quad j = 0, 1, \ldots, n-1, \qquad a = \delta_1 < \delta_2 < \cdots < \delta_L = b. \tag{2}$$

Here $f(x)$, $\alpha(x)$, $\beta_k(x)$, $\gamma_k(x)$, $K_{V,k}(x,t)$, $K_{F,k}(x,t)$ are known smooth enough functions and a_k, b_k, $A_{j,k,l}$, δ_l, d_j are appropriate constants. This is the most general form of the generalized linear pantograph integro-differential equations. The neutral type is caused by the presence of the highest derivative on the right hand of the IDE.

* Correspondence to: State Institution "Institute of Technical Problems of Magnetism of the National Academy of Sciences of Ukraine", Industrialnaya St., 19, 61106, Kharkov, Ukraine.
E-mail address: sergiyreutskiy@gmail.com.

http://dx.doi.org/10.1016/j.cam.2015.10.013
0377-0427/

Integro-differential equation has been one of the principal tools in various areas of applied mathematics, physics and engineering [1]. So, a lot of analytical and numerical approaches have been proposed in literature to solve them. The methods that use analytical approximation are mainly based on the Adomian decomposition method, homotopy analysis method and their modifications, variational iteration method [2–6].

Many methods have been developed for numerical solving of IDEs by using a prescribed form of the solution as a series expansion over a system of basis functions. Then the problem is transformed into a system of algebraic equations using the collocation inside the solution domain. Depending on the basis functions, this group includes the Chebyshev and Taylor collocation methods [7–14], the Bessel methods proposed by Yüzbaşı et al. [15,16]. The Laguerre collocation method was applied for solving a class of the IDEs with linear functional arguments in [17]. Two-point mixed problems are considered. The Legendre spectral collocation method for neutral and high-order Volterra IDE proposed by Wei and Chen [18,19], the Hermite collocation method [20]. The Haar wavelet method [21–24], the sinc-collocation method [25] and the tau method [26,27] have recently been developed. The Legendre collocation matrix method was proposed in [28] to solve high-order linear Fredholm integro-differential equations under the mixed conditions. The Galerkin method using the Legendre basis functions for solving linear Fredholm integro-differential problems was proposed in [29]. In [30], Maleknejad and Attary studied application of the Shannon wavelets approximation for the numerical solution of linear Fredholm integro-differential equations.

Functional integro-differential equations are often used to model some problems with aftereffect in mechanics and the related scientific fields [31]. Different particular methods have been presented for numerical solutions of the mentioned problems. The approximate solution in terms of the shifted Legendre polynomials with unknown coefficients was proposed by Saadatmandi and Dehghan in [32] for solving the higher-order linear Fredholm integro-differential–difference equation with variable coefficients. Gülsu and Sezer proposed in [33] the Taylor collocation method for solving IDEs with linear functional arguments. The Taylor series method for general higher order linear Fredholm integro-differential–difference equations with variable coefficients was proposed in [34]. In [35] the method based on the matrix relations between the Bessel polynomials and their derivatives was proposed for solving general linear Fredholm integro-differential–difference equations. Wei and Chen proposed in [19] the method of the Legendre collocation discretization for IDEs with the neutral term in the equation. The operational collocation method with the shifted Jacobi polynomial bases is applied to approximate the solution of linear and nonlinear functional IDEs by Borhanifar and Sadri in [36]. The collocation method based on the Laguerre polynomials was proposed by Yüzbaşı to solve the pantograph-type Volterra IDEs in [37]. The method based on the Bernoulli polynomials and collocation procedure was presented in [38] to obtain the solution of the higher order linear Fredholm integro-differential–difference equations with the mixed conditions. The spectral method which uses the Lagrange interpolation procedure for solving pantograph IDEs was proposed in [39]. The spectral method based on the Legendre-collocation for neutral IDEs was developed in [40].

The method presented in the paper is a further development of the numerical technique proposed earlier in [41] for multipoint problems with ODE. The general scheme is as follows. Let us write the governing IDE in the form:

$$L[u] = F(x, u), \tag{3}$$

where $L[\ldots]$ denotes the main part of the differential operator IDE. Throughout the paper we consider the case $L[u] = u^{(n)}$, where n is the maximum order of the derivative in the IDE under consideration. The remaining parts of the IDE are joined on the right hand side of (3). Let us write the boundary conditions in the form:

$$B[u] = g, \tag{4}$$

where the number of the conditions in (4) corresponds to the order of the derivative in (3). Let $\varphi_m(x)$ be some system of basis functions on $[a, b]$ such that the right hand side of (3) can be approximated by the linear combination:

$$F(x, u) = \sum_{m=1}^{M} q_m \varphi_m(x). \tag{5}$$

Besides, we assume that for each $\varphi_m(x)$ there exists $\Phi_m(x)$ given in the explicit analytic form which satisfies the conditions:

$$L[\Phi_m] = \varphi_m, \qquad B[\Phi_m] = 0. \tag{6}$$

We replace the original equation (3) by the approximate equation

$$L[u] = \sum_{m=1}^{M} q_m \varphi_m(x) \tag{7}$$

and consider the exact solution u_M of (7) as an approximate solution of the original equation (3). From Eq. (6) it follows that any linear combination

$$v_M(x) = \sum_{m=1}^{M} q_m \Phi_m(x)$$