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Computational study of blood flow in microchannels

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ABSTRACT

Using the Theory of Interacting Continua (Mixture Theory), blood is modeled as a two-component mixture, namely, plasma and red blood cells (RBCs). The plasma is assumed to behave as a Newtonian fluid and the RBCs are modeled as a suspension of rigid spherical particles with a viscosity dependent on the shear-rate and the hematocrit. The drag and lift forces are implemented through RBC–plasma interaction forces. We solve the governing equations using the OpenFOAM, an open source CFD code for two phase flow simulations. The two-phase simulations predict RBC depletion in the corner of a sudden expansion channel. The RBC depletion length is found to increase with decreasing the flow rate and the hematocrit. There is a qualitatively good agreement between the simulation results and the experimental data.

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1. Introduction

Blood is a suspension of red blood cells (RBCs), white blood cells (WBCs) and platelets in plasma. The hematocrit (Hct), or the RBC volume fraction, is about 45% of the (whole) human blood [1]. In large vessels (diameter greater than 1 mm) blood is assumed to be a homogeneous Newtonian fluid [2]. However, when the vessel's diameter is comparable to the characteristic size of the blood cells (i.e., 20–500 μm), blood is known to behave as a non-homogeneous fluid, exhibiting several non-Newtonian phenomena [3], including the Fahraeus effect [4], Fahraeus–Lindqvist effect [5], shear-thinning [1], spatially inhomogeneous distributions of RBCs [6–8], plasma-skimming [9], platelet deposition [10], etc. These micro-rheological phenomena have motivated investigators to develop two phase models for blood in order to study the spatial distributions of the RBCs, relative velocity and other related transport phenomena.

In this paper we use the mixture theory formulation of Massoudi et al. [11] to study the changes in the velocity and volume fraction profiles which occur due to different coefficients of drag and lift forces. This model assumes that blood can be considered a two-component mixture composed of RBCs suspended in plasma, i.e. a mixture of a viscous fluid infused with solid-like particles. Mixture Theory or the Theory of Interacting Continua was first presented within the framework of continuum mechanics by Truesdell [12,13]. It is a means of generalizing the equations and principles of the mechanics of a single continuum to include any number of continua. More detailed information, including an account of the historical development, is available in the articles by Atkin and Craine [14], Bowen [15], and in the book by Rajagopal and Tao [16]. Mixture theory is a homogenization approach where each phase is considered to be a single continuum and at each instant of time, every point in space is occupied by a particle belonging to each component of the mixture. Mixture Theory has been widely applied in various industrial applications such as fluidized beds, hydraulic transport, and coal combustion [17]. It

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Nomenclature

a	Acceleration vector
b	Body force vector
D	Symmetric part of the velocity gradient
I	Identity tensor
L	Gradient of velocity vector
p	Pressure
T	Stress tensor
v	Velocity vector
W	Spin tensor
x	Position vector

Greek letters

μ	Coefficient of viscosity
ρ	Density
ϕ	Volume fraction of red blood cells (the hematocrit)

Subscripts

1	Referring to the constituent 1 ($\rightarrow$ plasma)
2	Referring to the constituent 2 ($\rightarrow$ red blood cells)

Superscripts

T	Transpose

has also been used in a variety of biomechanics applications such as blood flow, growth in biological tissues, swelling and deformation of articular-cartilage, tumor growth, and RBCs suspended in plasma (see [18,19] and the references therein).

Using the Theory of Interacting Continua (Mixture Theory), blood is modeled as a two-component mixture, namely, plasma and red blood cells (RBCs). The plasma is assumed to behave as a Newtonian fluid and the RBCs are modeled as a suspension of rigid spherical particles with a viscosity dependent on the shear-rate and the hematocrit. The drag and lift forces are implemented through RBC–plasma interaction forces. We develop a two-phase CFD solver for blood flow using the OpenFOAM to investigate the spatial distribution of the RBCs in two microchannels and to study the effect of the RBC volume fraction and the flow rate on RBC flow behaviors. The two-phase simulations predict RBC depletion in the corner of a sudden expansion channel. The RBC depletion length is found to increase with decreasing the flow rate and the hematocrit. There is a qualitatively good agreement between the simulation results and the experimental data.

Section 2 provides a brief review of the governing equations of Mixture Theory. Section 3 describes the associated modeling of the stress tensors and the interaction forces. The governing equations of motion are then discretized and solved using OpenFOAM, an open source CFD code, as described in Sections 4 and 5.

2. A brief review of mixture theory

The plasma component is represented by S_1 and the RBC component by S_2. At each instant of time, t, it is assumed that each point in space is occupied by particles belonging to both S_1 and S_2. Let $\mathbf{X}_1$ and $\mathbf{X}_2$ denote the positions of S_1 and S_2 in the reference configuration. The motions of the constituents are represented by the mappings (χ_1 and χ_2) (see [15]):

$$\mathbf{x}_1 = \chi_1(\mathbf{X}_1, t), \quad \text{and} \quad \mathbf{x}_2 = \chi_2(\mathbf{X}_2, t). \tag{1}$$

These motions are assumed to be one-to-one, continuous, and invertible. The kinematical quantities associated with these motions are

$$\begin{aligned} &\mathbf{v}_1 = \frac{d_1\chi_1}{dt}, \quad \mathbf{v}_2 = \frac{d_2\chi_2}{dt}, \quad \mathbf{a}_1 = \frac{d_1\mathbf{v}_1}{dt}, \quad \mathbf{a}_1 = \frac{d_2\mathbf{v}_2}{dt}, \quad \mathbf{L}_1 = \frac{\partial\mathbf{v}_1}{\partial\chi_1}, \quad \mathbf{L}_2 = \frac{\partial\mathbf{v}_2}{\partial\chi_2} \\ &\mathbf{D}_1 = \frac{1}{2}\left(\mathbf{L}_1 + \mathbf{L}_1^T\right), \quad \mathbf{D}_2 = \frac{1}{2}\left(\mathbf{L}_2 + \mathbf{L}_2^T\right), \quad \mathbf{W}_1 = \frac{1}{2}\left(\mathbf{L}_1 - \mathbf{L}_1^T\right), \quad \mathbf{W}_2 = \frac{1}{2}\left(\mathbf{L}_2 - \mathbf{L}_2^T\right) \end{aligned} \tag{2}$$

where **v** denotes the velocity vector, **a** the acceleration vector, **L** the velocity gradient, **D** the symmetric part of the velocity gradient, and **W** the spin tensor. d_1/dt denotes differentiation with respect to t, holding $\mathbf{X}_1$ fixed, and d_2/dt denotes the same operation holding $\mathbf{X}_2$ fixed.