



Mathematical analysis and finite element simulation of a magnetized ferrite model[☆]



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ABSTRACT

Microwave scattering by obstacles made of ferrite materials has been studied for a long time because of its important applications in radar, satellite, optical imaging system, signal processing, atmospheric science, and even cloaking device design. In this paper, we present for the first time the rigorous mathematical analysis of a magnetized ferrite model widely used by engineers. The well-posedness of the model is established first. Then we develop a finite element time-domain method for solving this model. Numerical stability similar to the continuous stability is proved for our scheme, and the optimal error estimate in the L_2 norm is obtained. Numerical results are consistent with our theoretical analysis and show how wave propagates in the ferrite material.

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1. Introduction

Ferrite materials are very popular in microwave applications due to their many interesting characteristics such as frequency tunability, polarization diversity, and signal control (cf. [1]). In microstrip patch antenna design, ferrite substrates are often used instead of the normal dielectric material, since they can reduce the radar cross section of the antennas, and change the beam width and direction in ferrite-based antenna arrays. Due to the inherent anisotropy and nonlinearity of the ferrite materials, finding analytical solutions for ferrite based problems is often impossible, and development of accurate and efficient numerical methods plays a very important role. However, most publications on simulation of wave propagation in ferrites (e.g., [1–4]) are based on the finite difference time-domain (FDTD) methods due to its simplicity. It is well-known that the FDTD methods suffer when the underlying problem has a complicated geometry. In overcoming this disadvantage, there are some developments of finite element method (FEMs) for solving the ferrite model in engineering community. For example, Brown et al. [5] developed a finite element frequency-domain method to solve the time-harmonic Maxwell's equations for simulating patch antennas on ferromagnetic substrates. Zhou and Davis [6] used FEMs with edge elements for modeling waveguides loaded with magnetized ferrite. However, most publications of FEMs on ferrites are based on frequency-domain.

In this paper, based on our previous works on developing finite element time-domain (FETD) methods for solving wave propagation in metamaterials (e.g., [7–10]), we investigate the popular magnetized ferrite model in time-domain from the mathematical point of view. Of course, there exist many excellent results on the existence, uniqueness and FEMs for solving

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Maxwell's equations in other media (e.g., [11–27] and references therein). The mathematical analysis and development of our FETD method for solving the ferrite model is original.

The paper is organized as follows. In Section 2, we first present the governing equations for the popular magnetized ferrite model used by many engineers [1,4]. Then we establish a continuous stability for the model, and prove the existence and uniqueness of the solution to the modeling equations. In Section 3, we develop a FETD method for solving the ferrite model. Numerical stability similar to the continuous case is obtained first. Following the similar technique, the optimal error estimate in the L_2 norm is proved. In Section 4, numerical results with an exact solution case and a practical wave propagation problem in ferrite are presented. We conclude the paper in Section 5.

2. The modeling equations

To simulate the response of a microstrip patch antenna printed on a magnetized ferrite substrate, the Polder frequency dependent permeability tensor is often adopted (cf. [1,4]):

$$\mu(\omega) = \begin{bmatrix} \mu_{xx} & \mu_{xy} & 0 \\ -\mu_{xy} & \mu_{yy} & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \quad (2.1)$$

where it is assumed that the magnetizing Direct Current (DC) field is in the z -direction. The components in (2.1) are

$$\mu_{xx} = \mu_{yy} = \mu_0 \left[1 + \frac{\omega_m(\omega_0 + j\omega\alpha)}{(\omega_0 + j\omega\alpha)^2 - \omega^2} \right] = \mu_0 + \frac{\mu_0\omega_m\omega_0 + (j\omega)\alpha\mu_0\omega_m}{\omega_0^2 + (j\omega)2\alpha\omega_0 + (j\omega)^2(1 + \alpha^2)} \quad (2.2)$$

and

$$\mu_{xy} = \frac{j\omega\mu_0\omega_m}{(\omega_0 + j\omega\alpha)^2 - \omega^2} = \frac{(j\omega)\mu_0\omega_m}{\omega_0^2 + (j\omega)2\alpha\omega_0 + (j\omega)^2(1 + \alpha^2)} \quad (2.3)$$

where $j = \sqrt{-1}$, μ_0 is the free space permeability, ω_m is the magnetization frequency, ω_0 is the resonance frequency, $\alpha \geq 0$ is the damping/loss factor, and ω is the general wave frequency. A lossless case (i.e., $\alpha = 0$) was considered in [5].

Using the constitutive equation $\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \mu \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$ and the $\exp(j\omega t)$ time variance, we can obtain the time-domain governing equations (cf. [1])

$$\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H}, \quad (2.4)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (2.5)$$

$$\begin{aligned} & \mu_0(1 + \alpha^2) \frac{\partial^2}{\partial t^2} \begin{pmatrix} H_x \\ H_y \end{pmatrix} + \mu_0\alpha(2\omega_0 + \omega_m) \frac{\partial}{\partial t} \begin{pmatrix} H_x \\ H_y \end{pmatrix} + \mu_0\omega_0(\omega_0 + \omega_m) \begin{pmatrix} H_x \\ H_y \end{pmatrix} \\ & + \begin{pmatrix} 0 & \mu_0\omega_m \\ -\mu_0\omega_m & 0 \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \\ & = (1 + \alpha^2) \frac{\partial^2}{\partial t^2} \begin{pmatrix} B_x \\ B_y \end{pmatrix} + 2\alpha\omega_0 \frac{\partial}{\partial t} \begin{pmatrix} B_x \\ B_y \end{pmatrix} + \omega_0^2 \begin{pmatrix} B_x \\ B_y \end{pmatrix}, \end{aligned} \quad (2.6)$$

$$B_z = \mu_0 H_z. \quad (2.7)$$

To simplify the notation, from now on we replace $\mu_0 \mathbf{H}$ by a new variable, which is still denoted as $\mathbf{H}$ without confusion. Moreover, we will denote time derivatives $\frac{\partial u}{\partial t}$ and $\frac{\partial^2 u}{\partial t^2}$ as u_t and u_{tt} , respectively. To facilitate the analysis of the complex system (2.4)–(2.7), we combine the constitutive equation (2.6)–(2.7) into one vector equation, and arrange the system (2.4)–(2.7) in a different order as follows:

$$\mathbf{B}_t = -\nabla \times \mathbf{E}, \quad (2.8)$$

$$(1 + \alpha^2) \mathbf{H}_{tt} + M_a \mathbf{H}_t + \omega_0(\omega_0 + \omega_m) \mathbf{H} = (1 + \alpha^2) \mathbf{B}_{tt} + M_b \mathbf{B}_t + M_c \mathbf{B}, \quad (2.9)$$

$$\epsilon_0 \mu_0 \mathbf{E}_t = \nabla \times \mathbf{H}, \quad (2.10)$$

where the matrices M_a , M_b and M_c are

$$M_a = \alpha(2\omega_0 + \omega_m) I_3 + \begin{pmatrix} 0 & \omega_m & 0 \\ -\omega_m & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.11)$$

$$M_b = \begin{pmatrix} 2\alpha\omega_0 & 0 & 0 \\ 0 & 2\alpha\omega_0 & 0 \\ 0 & 0 & \alpha(2\omega_0 + \omega_m) \end{pmatrix}, \quad M_c = \begin{pmatrix} \omega_0^2 & 0 & 0 \\ 0 & \omega_0^2 & 0 \\ 0 & 0 & \omega_0(\omega_0 + \omega_m) \end{pmatrix}. \quad (2.12)$$

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