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Haar wavelet method for some nonlinear Volterra integral equations of the first kind

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HIGHLIGHTS

- Solving some nonlinear first kind Volterra integral equations using Haar wavelet.
- Haar wavelet method is simple and fast.
- Solved examples demonstrate the accuracy and efficiency of the present method.
- Present method gives better results than the numerical methods described in past.
- Necessary accuracy may be obtained by increasing the number of calculation points.

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ABSTRACT

We present here a simple efficient Haar wavelet method for numerical solution of a class of nonlinear Volterra integral equations of the first kind. The present method is based on converting nonlinear Volterra integral equations of the first kind into linear Volterra integral equations of the second kind. Numerical examples are given to illustrate efficiency and accuracy of the present method. Comparison with numerical methods in the past has also been done.

1. Introduction

In literature, many numerical methods using wavelets have been presented for solving nonlinear Volterra integral equations of the first kind. In [1–3], numerical methods based on collocation and implicit Runge–Kutta were described for the solution of first and second kind Volterra integral equations. Methods based on product integrations were described in [4–8] for solution of second kind Volterra integral equations with singular, nonsingular and periodic kernels. Method described in [7] can be used directly for first kind Volterra integral equations. An operational Haar wavelet method for solving fractional Volterra integral equations is also presented in [9]. A survey of numerical methods for solving nonlinear integral equations have been presented in [10]. In mathematical physics and engineering, problems are often reduced to Volterra integral equations of the first kind. Therefore, we present here Haar wavelet method for solution of these equations. Volterra integral equations of the first kind are inherently ill-posed. Due to this, small variations in the problem can cause very large

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variations in the solution obtained, see, for example [11–13]. From past literatures, we know that it is more difficult to find the numerical solutions of ill-posed problems. Unbounded errors are obtained due to small errors in observations of data of ill-posed problems arising in science, engineering, medicines and ecology. In [11,14] different regularization methods have been proposed to overcome with the difficulty of ill-posed problems.

Consider the nonlinear Volterra integral equation of the form

$$\int_a^x K(x,t)S(y(t))dt = g(x), \quad x \in [a,b], \tag{1}$$

where K, S and g are the given smooth functions and S is invertible and nonlinear in y. The solution y is determined under the assumption that $g(a) = 0$. As an illustration, some forms of $S(y(t))$ are given below:

- $y^{(n)}(t)$, it is assumed that $y = y^{(1)} = y^{(2)} = \cdots = y^{(n-1)} = 0$ at $t = a$, where $y^{(n)}$ represents the nth derivative of y with respect to t,
- $y^n(t)$, where y^n represents the nth power of $y(t)$,
- $\ln(y(t))$ and $\ln(\sqrt{y(t)})$,
- $\sqrt{y(t)}$ and $\sqrt{\sqrt{y(t)}}$,
- $\sqrt{\sin(y(t))}$ and $\sqrt{\cos(y(t))}$.

Equations of the form (1) have been studied in [15–18]. Babolian et al. [15,16] suggested direct method for solving Volterra integral equation of the first kind using operational matrix with block pulse functions and operational matrices of piecewise constant orthogonal functions respectively. Biazar et al. [17,18] described a method for solving nonlinear Volterra integral equation of the first kind using Adomian decomposition method and homotopy perturbation method. Lepik [19,20] have described methods for solution of integral and differential equations using Haar wavelets. Chen and Hsiao [21] have described numerical methods for solving lumped and distributed parameter systems using Haar wavelets.

In Section 2, we briefly describe Haar wavelet method. In Section 3, we have given function approximation. In Section 4, error analysis is presented. In Section 5, we propose the method for solving nonlinear Volterra integral equations of the first kind using Haar wavelet and in Section 6, numerical examples have been solved using the present method to illustrate the efficiency and accuracy of present method. In this section, we also compare the present method with numerical methods presented in [15,16].

2. Haar wavelet method

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0, 1] as below:

$$h_i(x) = \begin{cases} 1, & \alpha \le x < \beta, \\ -1, & \beta \le x < \gamma, \\ 0, & \text{elsewhere}, \end{cases} \tag{2}$$

where $\alpha = \frac{k}{m}$, $\beta = \frac{k+0.5}{m}$ and $\gamma = \frac{k+1}{m}$. Integer $m = 2^j$, $(j = 0, 1, 2, 3, 4, \ldots, J)$ indicates the level of the wavelet, and $k = 0, 1, 2, 3, \ldots, m-1$ is the translation parameter. Maximal level of resolution is J. The index $i = m + k + 1$. In case of minimal values, $m = 1$, $k = 0$ we have $i = 2$. The maximal value of i is $i = 2M$, where $M = 2^J$. It is assumed that for value $i = 1$, the corresponding scaling function in [0, 1] is as:

$$h_1(x) = \begin{cases} 1, & 0 \le x \le 1, \\ 0, & \text{elsewhere}. \end{cases} \tag{3}$$

Let us define the collocation points $x_l = \frac{(l-0.5)}{2M}$, where $l = 1, 2, 3, \ldots, 2M$ and discretize the Haar function $h_i(x)$. The Haar coefficient matrix is defined as $HAAR(i,l) = (h_i(x_l))$, which is a square matrix of order $2M \times 2M$. The Haar coefficient matrix of order 8 is given below:

$$HAAR_8(x) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}. \tag{4}$$

The operational matrix of integration, which is a $2M \times 2M$ square matrix, is defined by the relations:

$$P_{i,1}(x) = \int_0^x h_i(t)dt, \tag{5}$$

$$P_{i,n+1}(x) = \int_0^x P_{i,n}(t)dt, \tag{6}$$