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Rigorous numerics for piecewise-smooth systems: A functional analytic approach based on Chebyshev series

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ABSTRACT

In this paper, a rigorous computational method to compute solutions of piecewise-smooth systems using a functional analytic approach based on Chebyshev series is introduced. A general theory, based on the radii polynomial approach, is proposed to compute crossing periodic orbits for continuous and discontinuous (Filippov) piecewise-smooth systems. Explicit analytic estimates to carry the computer-assisted proofs are presented. The method is applied to prove existence of crossing periodic orbits in a model nonlinear Filippov system and in the Chua's circuit system. A general formulation to compute rigorously crossing connecting orbits for piecewise-smooth systems is also introduced.

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1. Introduction

In this paper, we introduce a rigorous computational method for the study of piecewise-smooth (PWS) systems, which are described by a finite set of ODEs

$$\dot{u} = g^{(i)}(u), \quad u \in \mathcal{R}_i \subset \mathbb{R}^n \tag{1}$$

where $\mathcal{R}_1, \ldots, \mathcal{R}_N$ are open non-overlapping regions separated by $(n-1)$-dimensional manifolds $\Sigma_{ij} := \partial\mathcal{R}_i \cap \partial\mathcal{R}_j$ for $i \neq j$. When non empty, the set Σ_{ij} is the common boundary of the two adjacent regions $\mathcal{R}_i$ and $\mathcal{R}_j$, and we refer to it as a *switching manifold*. Given $\Sigma_{ij} \neq \emptyset$, assume the existence of $H^{(i,j)} : \mathbb{R}^n \to \mathbb{R}$ such that

$$\Sigma_{ij} = \left\{ u \in \mathbb{R}^n : H^{(i,j)}(u) = 0 \right\}. \tag{2}$$

Assume that the functions $g^{(i)}$ and $H^{(i,j)}$ are smooth, and that the union of all regions and switching manifolds covers the entire state space.

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Let us introduce some definitions, following closely the presentation of [1].

A PWS system is called *continuous* if, for all couple $i, j \in \{1, \ldots, N\}$ such that $\Sigma_{ij} \neq \emptyset$, $g^{(i)}(u) = g^{(j)}(u)$ at any point $u \in \Sigma_{ij}$, where in this case, we implicitly considered the continuous extension of the $g^{(i)}$'s to the closure of $\mathcal{R}_i$. For continuous PWS systems the tangent vectors $\dot{u}$ are uniquely defined at any point of the state space, and orbits in region $\mathcal{R}_i$ approaching transversally Σ_{ij}, cross it and enter into the adjacent region $\mathcal{R}_j$. Therefore, in continuous PWS systems, all orbits entering the switching manifold transversally undergo crossing. We refer to such orbits as *crossing orbits*.

The situation is different for discontinuous PWS systems, which are often called *Filippov systems*. In this case, two different tangent vectors $g^{(i)}(u)$ and $g^{(j)}(u)$ can be assigned to a point $u \in \Sigma_{ij}$. If the transversal components of $g^{(i)}(u)$ and $g^{(j)}(u)$ have the same sign, that is if

$$\left(\left(\nabla H^{(i,j)}(u)\right)^T \cdot g^{(i)}(u) \right) \left(\left(\nabla H^{(i,j)}(u)\right)^T \cdot g^{(j)}(u) \right) > 0, \tag{3}$$

then the orbit crosses the switching manifold Σ_{ij} with a discontinuity in its tangent vector at u. An orbit which visits some switching manifolds in a way that (3) holds at any point of the visited switching manifolds is also referred to as a *crossing orbit*. If on the other hand the transversal components of $g^{(i)}(u)$ and $g^{(j)}(u)$ have different signs at $u \in \Sigma_{ij}$, that is if

$$\left(\left(\nabla H^{(i,j)}(u)\right)^T \cdot g^{(i)}(u) \right) \left(\left(\nabla H^{(i,j)}(u)\right)^T \cdot g^{(j)}(u) \right) < 0, \tag{4}$$

then the two vector fields are pushing in opposite directions, and the solution remains on the switching manifold and slides on it for some time. While there are different ways of defining the motion of the solution on a switching manifold, the convexification method proposed by Filippov in [2] is perhaps the most natural. In the present paper, we do not discuss Filippov convexification's method and refer instead to [1–3] for details. The approach of Filippov leads to a classification of other type of orbits, namely *crossing and sliding orbits*, and *sliding orbits*. In this paper, we consider only crossing orbits.

An important class of crossing orbits in the study of PWS systems is given by *crossing periodic orbits* (CPOs), which are periodic orbits with isolated points in common with the switching manifolds they visit. Another important class of crossing orbits in the study of PWS systems is given by *crossing connecting orbits* (CCOs) (which connect two equilibria) with isolated points in common with the switching manifolds they visit.

The goal of this paper is to adapt the recently developed rigorous computational methods of [4–7] for the study of PWS systems, with a particular emphasis on the study of CPOs and CCOs. We expand the solutions using Chebyshev series, and we obtain computer-assisted proofs in a Banach space of fast decaying Chebyshev coefficients.

A rigorous computational method goes beyond a standard a posteriori analysis of numerical computations. More explicitly, the field of rigorous numerics aims at developing mathematical theorems formulated in such a way that the assumptions can be rigorously verified by a computer. The approach requires an a priori setup that allows analysis and numerics to work together: the choice of function space, the choice of the basis functions, the Galerkin projection, the analytic estimates, and the computational parameters must all work hand in hand to bound the errors due to approximation, rounding and truncation, and this needs to be sufficiently tight for the proof to go through.

The first step of our approach is to set up an equivalent formulation of the form $F(x) = 0$, where $F : X \to Y$ with X and Y two infinite dimensional Banach spaces, whose solution $x \in X$ corresponds to the targeted dynamical object of interest (in our case a CPO or a CCO). Setting up the operator F requires expanding the solution using a spectral Chebyshev method. The next step is to consider a finite dimensional Galerkin projection of F, to apply Newton's method on it and to obtain a numerical approximation $\bar{x}$ to a solution of $F(x) = 0$. We then construct, with the help of the computer, an injective approximate inverse A of $DF(\bar{x})$ so that $AF : X \to X$. We define a Newton-like operator $T : X \to X$ by $T(x) = x - AF(x)$, and we aim at obtaining

(a) the existence of $\tilde{x} \in X$ such that $T(\tilde{x}) = \tilde{x}$, or equivalently (since A is injective) such that $F(\tilde{x}) = 0$;
(b) the existence of an explicit and small $r > 0$ such that $\|\tilde{x} - \bar{x}\|_X \leq r$.

The existence of the solution $\tilde{x} \in X$ and of the explicit error bound r is obtained by applying a modified version of Newton–Kantorovich theorem, namely the radii polynomial approach. The radii polynomials provide an efficient mean of determining a closed ball $\overline{B_r(\bar{x})}$ of radius r centered at the numerical approximation $\bar{x}$ on which the Newton-like operator $T(x) = x - AF(x)$ is a contraction. We present carefully this whole process in general in the context of computing CPOs.

It is important to mention that this work is by no means the first attempt to study PWS systems within the field of rigorous numerics. A by now classical example that has been studied rigorously with the help of the computer is Chua's circuit system [8,9]. The existence of a homoclinic orbit for some unknown parameter value within a certain range of the Chua circuit was shown in [10], and existence of chaos was therefore obtained. In his study of the Chua's system, Galias introduced rigorous integration for piecewise-linear (PWL) systems [11–13]. He computed rigorously CPOs in [12], and *sliding periodic orbits* in [13]. Note that the Chua's circuit system is a continuous PWL systems, and therefore it is not a Filippov system.

Moreover, it is important to note that Chebyshev series have been used before to obtain computer-assisted proofs of existence of connecting orbits [5,14], of solutions of boundary value problems [15] and to study Cauchy problem [5].

While we focus our attention on the computation of CPOs and CCOs, a very similar approach could be developed for initial value problems and more general boundary value problems, as considered for instance in [5].

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