Journal of Computational and Applied Mathematics 290 (2015) 26–43

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Inexact Restoration method for nonlinear optimization without derivatives

Ma. Belén Arouxét [a], Nélida E. Echebest [a,*], Elvio A. Pilotta [b]

[a] Departamento de Matemática, FCE, Universidad Nacional de La Plata, CP 172, 1900 La Plata, Argentina

[b] Facultad de Matemática, Astronomía y Física, CIEM (CONICET), Universidad Nacional de Córdoba, Medina Allende s/n, Ciudad Universitaria (5000), Córdoba, Argentina

A R T I C L E I N F O

Article history:
Received 29 April 2014
Received in revised form 23 March 2015

MSC:
65K05
90C30
90C56

Keywords:
Inexact Restoration
Derivative-free optimization
Trust-region methods
Polynomial interpolation

A B S T R A C T

A derivative-free optimization method is proposed for solving a general nonlinear programming problem. It is assumed that the derivatives of the objective function and the constraints are not available. The new method is based on the Inexact Restoration scheme, where each iteration is decomposed in two phases. In the first one, the violation of the feasibility is reduced. In the second one, the objective function is minimized onto a linearization of the nonlinear constraints. At both phases, polynomial interpolation models are used in order to approximate the objective function and the constraints. At the first phase a derivative-free solver for box constrained optimization can be used. For the second phase, we propose a new method ad-hoc based on trust-region strategy that uses the projection of the simplex gradient on the tangent space. Under suitable assumptions, the algorithm is well defined and convergence results are proved. A numerical implementation is described and numerical experiments are presented to validate the theoretical results.

Contents

* Corresponding author.
E-mail addresses: belen@mate.unlp.edu.ar (M.B. Arouxét), opti@mate.unlp.edu.ar (N.E. Echebest), pilotta@famaf.unc.edu.ar (E.A. Pilotta).

http://dx.doi.org/10.1016/j.cam.2015.04.047

1. Introduction

We present a new method for solving the general nonlinear programming problem

$$\min f(x) \text{ subject to } x \in \Omega, \quad C(x) = 0, \tag{1}$$

where $\Omega = \{x \in \mathbb{R}^n | L \leq x \leq U,\ L < U\}, f : \mathbb{R}^n \to \mathbb{R},\ C : \mathbb{R}^n \to \mathbb{R}^m$, where the derivatives of the objective function and the constraints are not available, although we assume that all the functions are continuously differentiable.

This kind of problems appears in many real world situations. For instance, when the functional values are the results of physical measurements or when the calculation of analytical derivatives is impractical [1–3].

Several methods and algorithms were developed for the unconstrained and box-constrained cases [4–9]. Later, in the last decade, some methods for the linearly constrained optimization problems without derivatives were proposed [10–15]. Derivative-free methods for more general constraints were addressed by means of Augmented Lagrangian approaches in [16–18].

Following the ideas of Powell's methods (BOBYQA) [9], where polynomial interpolation and trust-region strategy were used for box-constrained derivative-free optimization, we propose a method for the general optimization problem. Our method is based on the Inexact Restoration (IR) approach introduced in [19] and revised in [20,21]. A survey on this subject can be found in [22]. Each iteration includes two different phases: restoration and optimization. In the Restoration phase, which is executed once per iteration, an intermediate point (restored point) is found such that its infeasibility is a fraction of the infeasibility of the current point.

At the Optimization phase, a trial point belonging to π_k, a linearization of the feasible region around the restored point, is computed such that the objective function value is lower than in the restored point. A Lagrangian function can be also used at the Optimization phase as it is proposed in [23,21]. By means of a merit function, the new iterate is accepted or rejected. In case of rejection, the trust-region radius is reduced and the Optimization phase is repeated around the same restored point. This method improves almost separately the infeasibility and optimality. Filter criterion could be used instead of using a merit function [24–27]. One of the more attractive features of the IR method is that its theory allows us to use any efficient algorithm to perform each phase.

Recently, Bueno–Friedlander–Martínez–Sobral [28] also proposed a method based on IR for solving a nonlinear derivative-free optimization problem in which the derivatives of the constraints are available. In our work, the derivatives of the objective function and the constraints are not available and we approximate them by polynomial models, which is one of the main differences with the previous cited work.

We have taken into account the flexibility that IR method provides for choosing different subalgorithms in each phase, and therefore we performed two implementations of our method using two different solvers for Restoration phase: BOBYQA [9] and TRB-Powell [4].

On the other hand, for the Optimization phase, a derivative-free optimization problem with linear constraints is formulated. This problem could be solved by any efficient solver for linearly constrained derivative-free optimization, as the method introduced by Kolda, Lewis and Torczon [13], however we formulated an algorithm ad-hoc.

The paper is organized as follows. In Section 2 we briefly describe the IR method [19]. In Section 3 we introduce our derivative-free algorithm (IR-DFO) and some preliminary theoretical results. Also, we prove that the new algorithm is well defined. Assuming suitable hypotheses we analyze some global convergence results in Section 4. Implementation details and numerical experiments are shown in Section 5. Finally, some conclusions are made in Section 6.

Notation. Unless otherwise specified, our norm $\|\cdot\|$ is the standard Euclidean norm.

We let B denote a closed ball in $\mathbb{R}^n$ and $B(z; \Delta)$ denote the closed ball centered at z, with radius $\Delta > 0$.

e_i denotes the ith coordinate vector of $\mathbb{R}^n$.

We denote $C'(x) \in \mathbb{R}^{m\times n}$, the Jacobian matrix of $C(x)$ and $C_j'(x) = \nabla C_j(x)^T$ for $j = 1, \ldots, m$.

2. Inexact Restoration methods

In this section we give a description of the IR method [19] along with some preliminary definitions.

First of all, we define a measure of infeasibility given by: $h(x) = \|C(x)\|$. We used a penalty-like nonsmooth merit function, which combines feasibility and optimality, to measure the progress to the solution. This function is given by

$$\psi(x, \theta) = \theta f(x) + (1 - \theta)h(x), \tag{2}$$

where $\theta \in [0, 1]$ is a penalty parameter used to give different weights to the objective function and the measure of infeasibility. The choice of the parameter θ at each iteration depends on practical and theoretical considerations. See [19].

Given $y^k \in \mathbb{R}^n$ we define a linear approximation of the feasible region of (1) as

$$T(y^k) = \{x \in \Omega | C'(y^k)(x - y^k) = 0\}. \tag{3}$$

Moreover, given $z \in \mathbb{R}^n$ we also define $d_c(z)$ the projected direction of $-\nabla f(z)$ onto $T(z)$ as

$$d_c(z) = P_{T(z)}(z - \nabla f(z)) - z, \tag{4}$$

where $P_{T(z)}(w)$ denotes the orthogonal projection of w onto $T(z)$. A feasible point z such that $d_c(z) = 0$ is considered as a stationary point of (1) [29].