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Numerical modeling and optimization of the cryosurgery operations

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ABSTRACT

Numerical computation of the three dimensional problem of the freezing interface propagation during the cryosurgical impact coupled with the multi-objective optimization mechanism is used in order to improve the efficiency and safety of the cryosurgery operations performing. The heat transfer in soft tissue during the thermal exposure to low temperature is described by the Pennes bioheat model. The finite volume method combined with the control volume approximation of the heat fluxes is applied for the cryosurgery numerical modeling on the tumor tissue of a quite arbitrary shape. The flux relaxation approach is used for the stability improvement of the explicit finite difference schemes. The generalized method of effective traversals is proposed for the searching of the Pareto front segments as the multi-objective optimization problem solutions.

1. Introduction

Cancer incidence is one of the most important problems [1,2] at present time. The accelerating growth of cancer statistics has made many researchers to focus their studies on the developing and introducing more efficient and safe cancer treating mechanisms. Today cryosurgery is one of the widespread techniques of cancer treatment. In particular, cryoablation of the undesired tissue shows good medical results in prostate cancer treatment [3,4].

The cryosurgical treatment is known to be based on the tissue necrosis effect. The last one is caused by the ice ball propagation from the tips of cryoneedles in the tissue to treat. The cryosurgery operation performing on example of the prostate cancer cryosurgery treating is illustrated in detail in Fig. 1 (from left to right: cryoneedles, schematic and real operation performing, the ice-ball propagation from the cryoneedles in the prostate region). Unfortunately, real operations of cryosurgery are quite difficult in performing. Earlier the lack of knowledge about processes arising in biological tissues during the cryoimpact had been forcing surgeons to rely on their own experience and luck. Certainly, it should be noted that such approach had been leading to injury of healthy tissues and even to a lethal effect in some cases.

The aim of the majority of recent researches is the developing of a computerized planning tool for prediction and optimization of the cryosurgical impact results [5,6]. The last one is concerned with the impact optimization based on the so-called "bubble-packing" method, which may seem a quite heuristic or not much physically reasoned.

The current paper is concerned with the problem under investigation from the point of the explicit influence of the operation performing parameters on the physical processes taking place in the living tissue during the cryosurgery operations. Since there is no explicit relation between the performing parameters and the criteria of the operation success the mathematical modeling instrumentary is applied to realize the computational function which makes the transformation

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of the operation parameters on the efficiency criteria vector. The impact efficiency in current study is considered to be measured in the number of undestructed cells in tumor tissue on the one hand and in the number of injured cells in healthy tissue on the other hand after the operation is over. The realized function serves a role of the objective function in the multi-objective optimization problem [7,8]. The generalized effective traversals method is proposed as the algorithm for the Pareto's front [9] searching to determine optimal performance parameters.

An explicit scheme based on the finite volumes method is used for the numerical modeling of cryosurgery and is coupled with optimization algorithm. Classical Pennes model is applied to describe the bioheat transfer processes taking place in the living tissues. Since the proposed method of the performing parameters optimization supposes the multiple repeating of the cryosurgery modeling computations, the flux relaxation approach is involved for the stability improvement of explicit finite difference scheme. The last one leads to a decreasing of the computations time by taking a higher time steps. The optimal flux relaxation parameter is being determined for each grid scale and desirable time step according to the criterion of 3% accuracy on the problems with the well known exact solutions.

2. Classical bioheat transfer model

Heat transfer processes in living tissues in most of the recent works have usually being described by classical Pennes's bioheat transfer model (2.1):

$$\begin{aligned} &\rho C\frac{\partial T}{\partial t} = -\mathrm{div}\,\mathbf{Q} + \rho_b\omega_b C_b(T_b - T) + q_{met}, \quad \mathbf{Q} = -\kappa\nabla T, \\ &t > 0, \quad \mathbf{r} \in H \subset R^3, \end{aligned} \tag{2.1}$$

where T, ρ, C, κ are the tissue temperature, density, volumetric specific heat and thermal conductivity correspondingly, $\mathbf{r} = (x, y, z)$ is the vector of Cartesian coordinates, t is time. H is the healthy tissue region, computation area. U is the tumor tissue region assumed to be located inside the healthy tissue, i.e. $U \subset H$ (see Fig. 2). The second term known as the Pennes term in the right side responds to the thermal exchange between tissue and blood perfusing the tissue. Here ρ_b, C_b, ω_b are the blood density, volumetric specific heat and perfusion rate correspondingly. T_b is the temperature of blood entering the thermally-treated area. Many experimental studies have shown that the term q_{met}, corresponding to metabolic heat generation occurring due to oxidative–restorative reactions, which take place when the perfusing blood delivers an oxygen to the tissue cells, can be neglected during the cryosurgery modeling.

An enthalpy method [10] is used because according to the experimental studies a phase change in living tissues is blurred in a temperature range between $T_u = -1$ °C, upper boundary, and $T_l = -8$ °C, lower boundary. Hence, combination ρC is determined in the following manner:

$$\begin{aligned} &\rho C(T \le T_l) = \rho C|_{frozen}, \\ &\rho C(T_l \le T \le T_u) = \frac{1}{2}\left(\rho C|_{frozen} + \rho C|_{unfrozen}\right) + \frac{\rho|_{frozen} + \rho|_{unfrozen}}{4\triangle}L, \\ &\rho C(T \ge T_u) = \rho C|_{unfrozen}. \end{aligned} \tag{2.2}$$

In this way the phase change process is proved to be included in the governing equation (2.1) avoiding the use of computational methods suspecting the phase change interface tracking.

To completely define the problem to be solved the boundary and initial conditions must be determined:

$$T(t = 0, \mathbf{r}) = T_{body}. \tag{2.3}$$

Here T_{body} is the equilibrium body's temperature. In current study T_{body} is taken to be equal to 37 °C. Assuming the computation area's boundary Γ to be enough distant from the treating region we can consider no thermal fluxes to take place on the computation boundary, so boundary conditions can be defined as follows:

$$\mathbf{Q}(t, \mathbf{r} \in \Gamma) = 0. \tag{2.4}$$

Cryoprobes in current study are considered as point sources characterized by constant temperature and located in thermally treated region. In that way we have an additional condition on temperature field in cryotips' locations:

$$T(t, \mathbf{r} = \mathbf{R}_i) = T_{source}. \tag{2.5}$$

Here $\mathbf{R}_i$ is the position of ith cryoprobe, $i = 1, \ldots, N$. The source temperature is assumed to be maintained at −150 °C in current study.

3. Numerical computation

A finite volume approach is used to obtain an explicit scheme for numerical modeling of the tissue freezing problem. Let us replace the computation area by the indexed computation grid which consists of the arbitrary convex hexahedrons. Let each hexahedron (grid cell) be determined by 3 indexes i, j, k. After integrating the governing equation (2.1) upon the ijkth