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A state-space description for perfect-reconstruction wavelet FIR filter banks with special orthonormal basis functions

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ABSTRACT

This paper presents a state space description for wavelet FIR filter banks with perfect reconstruction using special orthonormal basis functions. The FIR structure guarantees the BIBO stability, robustness and improves the filter divergence problem while orthonormal basis functions have characteristics that make them attractive in the modeling of dynamic systems. The state space description presented in this paper has all of those advantages and is minimal.

1. Introduction

Dynamic system models are important to study problems that arise in closed-loop systems, systems in which conditions are converted into information that can be observed and controlled [1]. The modeling of real systems is of great importance to engineers, since models are usually needed for the design of new processes and the analysis of existing ones. Generally, advanced techniques of controller design, optimization and supervision are based on process models, and the quality of the model directly influences the quality of the final solution to the problem [2–4].

Several techniques are proposed for modeling dynamic systems. There are models obtained by using orthogonal functions that form a complete basis for the Lebesgue $L_2[0, \infty)$ space and orthonormal basis functions [2,3]. Such models have some characteristics that make them attractive for dynamic systems modeling: absence of output recursion, not requiring prior knowledge of the exact structure of the vector of regression; possibility to increase the capacity of representation of the model by increasing the number of orthonormal functions employed; natural uncoupling of the outputs in multivariable models; tolerance to unmodeled dynamics, and others [3].

On the other hand, the FIR (Finite Impulse Response) structure does not only guarantee both BIBO (Bounded Input/Bounded Output) stability and robustness to some parameter changes, but also improves the filter divergence problem [5–7].

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This paper presents a state space description for an orthonormal basis of functions developed in [8], which are used as wavelet FIR filter banks with perfect reconstruction. This description holds the advantages presented by orthonormal basis functions and the FIR structure, culminating in a minimal realization.

The contribution of this paper consists in developing a realization in state space for a wavelet filter bank, with the explicit presence of parameters that can be freely adjusted keeping the guarantees of perfect reconstruction and orthogonality. For this purpose, the parameterization described in [8] is adopted as a starting point. The novelty of the present work with respect to [8] is the proposal of a realization of the resulting filter in the state space.

Other studies about state space using FIR filters present some properties in common in relation to the proposed parameterization, and other aspects that can be checked in [9–13,5–7]. Unlike these studies, this work employs orthonormal wavelet filter banks, rather than FIR filters in a general way. An advantage of the wavelet parameterization in the state space is the presence of a certain number of free parameters that can be adjusted by the designer.

Within the scope of possible applications and extensions of the present paper, it is worth mentioning some representative contributions about innovative filtering and auxiliary model based estimation methods [14], as well as identification of dual-rate systems [15,16] using the hierarchical identification principle. Identification methods for a Hammerstein system with its linear dynamic subsystem being an observer canonical state space model were exploited in [17], with an approach that can effectively solve the identification problem of a class of bilinear identification systems. A discussion about parameter estimation algorithm to establish the mathematical models for dynamic systems and an estimated state based on recursive least squares algorithm can be found in [18]. A study considering modeling and identification problems for linear systems based on canonical state space models with d-step state-delay is given in [19]. A study about state filtering and parameter estimation problems for state space systems with scarce output availability using least squares based algorithm and an observer based parameter estimation algorithm is developed by [20]. About reconstruction, some related issues of non-uniformly sampled systems, including model derivation, controllability and observability, computation of single-rate models with different sampling periods, reconstruction of continuous-time systems, and parameter identification of non-uniformly sampled discrete-time systems are discussed in [21]. These works can link the results of this paper to other possible extensions.

2. Background

Sherlock and Monro [8] developed a formulation to parameterize the space of orthonormal wavelets by using a set of angular parameters, adapting the work about factorization of paraunitary matrices of [4]. This formulation is presented below.

Consider the low-pass filter analysis in a two-channel orthonormal filter bank with $2N$ coefficients $\{h_i\}$ and its z-transform

$$H(z) = \sum_{i=0}^{2N-1} h_i z^{-i} = H_0(z^2) + z^{-1} H_1(z^2),$$

where H_0 and H_1 are the polyphase components of $H(z)$, namely,

$$H_0(z) = \sum_{i=0}^{N-1} h_{2i} z^{-1} \quad \text{and} \quad H_1(z) = \sum_{i=0}^{N-1} h_{2i+1} z^{-1}. \tag{1}$$

In [4], Vaidyanathan proposed the factorization of paraunitary matrices $H_p(z)$ in the following manner

$$\begin{aligned} H_p(z) &= \begin{pmatrix} H_0(z) & H_1(z) \\ G_0(z) & G_1(z) \end{pmatrix} \\ &= \begin{pmatrix} C_0 & S_0 \\ -S_0 & C_0 \end{pmatrix} \prod_{i=1}^{N-1} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} C_i & S_i \\ -S_i & C_i \end{pmatrix}, \end{aligned} \tag{2}$$

where $C_i = \cos(\alpha_i)$, $S_i = \sin(\alpha_i)$ and $G_0(z)$ and $G_1(z)$ are the polyphase components of the high-pass filter analysis $G(z)$. This factorization generates all perfect reconstruction two-channel orthonormal filter banks with $2N$ coefficients, i.e., any filter can be written in terms of N parameters $\alpha_i \in [0, 2\pi)$. However, in order to the filter bank be built in terms of these parameters correspond to a basis of orthonormal wavelets, it is necessary that

$$\sum_{i=0}^{2N-1} h_i = \sqrt{2}. \tag{3}$$

The formulation of Sherlock and Monro [8] is developed from (2). In fact this formulation consists in rewriting (2) in a recursive form expressing the polyphase matrices corresponding to filters with length $2(N+1)$ in terms of the polyphase matrices corresponding to the filters of length $2N$

$$H_p^{(N+1)}(z) = H_p^{(N)}(z) \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} C_N & S_N \\ -S_N & C_N \end{pmatrix} \tag{4}$$