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On the ruin probability for nonhomogeneous claims and arbitrary inter-claim revenues

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ABSTRACT

Recently, Raducan et al. (2015) obtained recursive formulas for the ruin probability of a surplus process at or before claim instants under the assumptions that the claim sizes are independent, nonhomogeneous Erlang distributed, and independent of the inter-claim times (i.e., the times between two successive claims), which are assumed to be independent, identically distributed (i.i.d.), following an Erlang or a mixture of exponentials distribution. In this paper, we extend these formulas to the more general case when the inter-claim times are i.i.d. nonnegative random variables following an arbitrary distribution. We also present numerical results based on the new recursions, discuss some computational aspects and state a conjecture that connects the ordering of the claims arrival with the magnitude of the corresponding ruin probabilities.

1. Introduction

In this paper, we extend the recursive formulas obtained by Raducan et al. [1] for the ruin probability of a certain surplus process by generalizing its inter-claim times distribution. We shall deal with the ruin probability at or before a certain claim instant, which, as stated by Stanford and Stroinski [2], Stanford et al. [3], is interesting since it shows where the risk process is relatively vulnerable to ruin.

From a mathematical point of view, the surplus process can be modeled by considering two sequences of independent random variables (r.v.s): the positive claim sizes (CSs) $(X_n)_{n\geq 1}$ and the nonnegative inter-claim revenues (ICRs) $(Y_n)_{n\geq 1}$ (assuming that the premium is constant per unit time, we shall work with the inter-claim revenues, i.e., the premium difference between two consecutive claims, rather than with the inter-claim times). Then $\xi_n = X_n - Y_n$ represents the loss increment between the $(n-1)$th and the nth claim, and, by adding these loss increments, we obtain the total loss at each claim instant, which is involved in the evaluation of the ruin probability at or before the nth claim by means of the r.v. $L_{1,n} = \max\left(0, \xi_1, \xi_1 + \xi_2, \ldots, \sum_{i=1}^{n} \xi_i\right)$. More precisely, denoting this ruin probability by $\Psi_n(x)$ with x representing the initial capital, we have $\Psi_n(x) = \Pr(L_{1,n} > x)$. Therefore, the problem is to find the distribution of $L_{1,n}$.

In general, the usual assumptions under which the ruin probabilities related to this surplus model are studied are i.i.d. $(X_n)_{n\geq 1}$, independent of the i.i.d. $(Y_n)_{n\geq 1}$; in the classical model, $(Y_n)_{n\geq 1}$ are also exponentially distributed (for a review on ruin probabilities, see, e.g., the book by Assmussen and Albrecher [4]). However, there exist various attempts in relaxing the

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usual assumptions. For example, the classical Poisson process used to model the number of claims has been replaced with other processes, see, e.g., [3,5–11] etc. On the other hand, a dependency of the premium on the size of the surplus has been taken into account in [12].

Another attempt consists in relaxing the "identically distributed" (i.d.) condition imposed to the CSs, which yields a nonhomogeneous process. This is motivated by the tendency of increase of the CSs, phenomenon described in detail by Lefevre and Picard [13], and supported for example by the variation of the interest force, see also the survey by Paulsen [14]. However, such a non-i.d. assumption generates serious difficulties in the ruin probability evaluation, hence the related papers generally discuss recursions for the finite time ruin probability for discrete-time risk processes under restrictive assumptions, like: De Kok [15], who presented an approximate recursive algorithm based on the first two moments of the CSs distributions; Blazevicius et al. [16] obtained recursions for discrete and rational-valued CSs; or Castaner et al. [17], whose recursive algorithm needs the discretization of the CSs distributions.

Overcoming the discrete-time risk process and discrete CSs assumptions, Raducan et al. [1] also relaxed the i.d. condition on the CSs, paying the price that these claims are considered to be Erlang distributed, but with different parameters; they also considered two extensions of the Poisson claim number process where the i.i.d. inter-claim times follow an Erlang or a mixture of exponentials distribution, and they evaluated the ruin probability at or before a certain claim instant. In the current paper, we go further on, the novelty being that the ICRs are nonnegative i.i.d. r.v.s with an arbitrary distribution. The crucial result which makes possible to prove the recursions presented in Theorem 3.1 is Lemma 2.1; this lemma enables us to express the distribution of ξ_n by means of the Laplace transform of Y_n.

Therefore, the paper is structured as follows: in Section 2, we introduce some notation and we present Lemma 2.1 followed by some useful convolution corollaries. Then, in Section 3, we obtain the ruin probability exact formula and the recursions that are needed for the evaluation of the coefficients involved in this formula. Some particular cases are detailed in Section 3.2, among which the discrete-time process that can be easily obtained from our general surplus process. Some computational aspects related to the obtained formulas are discussed and numerically illustrated in Section 4, which also contains a detailed study concentrated on the parameters influence on the ruin probability, and on the conditions that yield a certain order of these ruin probabilities. This section is divided into three subsections: the first one deals with i.i.d. CSs and discuss similar existing formulas in the literature; the second one presents several examples with non-i.d. CSs, also considering various distributions for the ICRs; based on these numerical examples, in the last subsection we propose a conjecture on the relation between the ordering of the claims arrival and the magnitude of the corresponding ruin probabilities.

The proofs are relegated in an Appendix.

2. Preliminaries

We denote by $\mathbb{R}^*$ the set of all real numbers without 0, by $\mathbb{N}^*$ the set of positive integers, and we let $\overline{1,n} = 1, \ldots, n$. By δ_x we denote the Dirac point measure defined, as usual, by $\delta_x(B) = 1_B(x)$ for any Borel set $B \subseteq \mathbb{R}$, where 1_B is the indicator function of B. By convention, $0! = 1$.

We use the same notation both for a distribution function (d.f.) and for the probability distribution generated by this d.f. on $\mathbb{R}$, i.e., if F is a d.f. and $B \subseteq \mathbb{R}$ is a Borel set, $F(B)$ means the probability of B, $F(x) = F((-\infty, x])$ and $\bar{F}(x) = F((x, \infty)) = 1 - F(x)$; if F denotes the d.f. of the r.v. X, we shortly write $X \sim F$.

We shall use the notation $\mathbf{e}_a$ for the exponential distribution with parameter $a > 0$, and $\mathbf{e}_a^n$ for the gamma distribution $Gamma(n, a)$, where $a > 0$ is the rate parameter and $n \in \mathbb{N}^*$ the shape parameter; in particular, this gamma distribution with positive integer-valued shape parameter is known under the name of Erlang. We recall that $\mathbf{e}_a^n(x) = 1 - e^{-ax} \sum_{i=0}^{n-1} \frac{(ax)^i}{i!}$ and its expected value is n/a.

If F is the d.f. of some r.v. Z, we denote by $F^{(+)}$ the d.f. of the positive part $Z_+ = \max(0, Z)$. Shortly, $Z \sim F \Rightarrow Z_+ \sim F^{(+)}$. Considering two d.f.s F and G and the real values a, b, it holds that

$$(aF + bG)^{(+)} = aF^{(+)} + bG^{(+)}. \tag{2.1}$$

From [18] or [19],

$$F^{(+)} = (1 - p)\,\delta_0 + pH, \tag{2.2}$$

where $p = \bar{F}(0) \in [0, 1]$ and $H = 1_{(0,\infty)}F$, i.e., H is a probability distribution on $(0, \infty)$ defined for any Borel set B by $H(B) = \frac{F(B \cap (0,\infty))}{\bar{F}(0)}$. To illustrate (2.2), we consider the following examples.

Example 2.1. Let $a, b > 0$ and F be the uniform distribution $U(-a, b)$. Then H is also a uniform distribution, $U(0, b)$, and $p = b/(a + b)$.

Example 2.2. Let $a > 0$ and $F \sim \mathbf{e}_a * \delta_{-1}$. Then, since for $x > 0$ we have on one hand, $\bar{F}(x) = e^{-a(x+1)} = e^{-a}e^{-ax}$ and, on the other hand, from (2.2), $\bar{F}(x) = p\bar{H}(x)$ with $p = e^{-a}$, it follows that $H \sim \mathbf{e}_a$.